Acknowledgement

UBC's Point Grey Campus is located on the traditional, ancestral, and unceded territory of the xʷməθkʷəy̓əm (Musqueam) people.

Course Information

Course Title	Course Code Number	Credit Value
Harmonic Analysis I	MATH 404/541	3

Time and Room: MWF 1pm-2pm. AUDX-Floor 1-Room 157 .

Prerequisites and corequisites

Prerequisites: MATH 300 (complex analysis), 68% or higher in MATH 321 (real variables II)
Corequisite: MATH 420 (real analysis I)

Contacts

Course Instructor	E-mail	Office Location	Office Hours
Pablo Shmerkin	pshmerkin@math.ubc.ca	MATH 210	By appointment

Other Instructional Staff

TA (Homework grader): Mukul Rai Choudhuri

Schedule of Topics

The list of topics and corresponding lectures is approximate and may be adjusted as the course progresses. Version of September 26, 2024.

1. Hardy-Littlewood maximal function, interpolation, and convolutions (Lectures 1-10).
   - Recap of L^p spaces and the Lebesgue integral.
   - The Hardy-Littlewood maximal function.
   - The Lebesgue differentiation theorem.
   - The Marcinkiewicz Interpolation Theorem.

- The Riesz-Thorin Interpolation Theorem
- Convolutions; Young's inequality.
- Approximations of the identity.

2. The Fourier transform and tempered distributions (Lectures 11-19)
   - Fourier transform on L^1.
   - The Schwartz space.
   - Fourier inversion.
   - Plancherel's Theorem and the Fourier transform on L^2.
   - Tempered distributions.
   - The uncertainty principle and the locally constant property for balls and ellipsoids.
3. Calderón-Zygmund theory (Lectures 20-26)
   - The Hilbert transform.
   - Calderón-Zygmund operators.
   - Calderón-Zygmund decomposition.
   - Weak type $(1, 1)$ estimates..

Learning Materials

You are expected to take notes during the lectures. We will not follow any textbook explicitly. There are many excellent books, among which:

1. T. Wolff. Lectures in Harmonic Analysis
   Available (with permission) here . Main source for the second part of the course (Fourier transform and tempered distributions).
2. L. Grafakos. Classical Fourier Analysis
   Available online through UBC library
3. Y. Katznelson. An introduction to Harmonic Analysis. 3rd Edition.
   Available online through UBC library
4. J. Duoandikoetxea. Fourier Analysis
   This book is closest to covering all topics of the course.

Assessments of Learning

There will be:

1. Five homework assignments to be handed in. These will be posted on Canvas. Homework solutions must be typeset in LaTex. Tentatively, the due dates are: September 27, October 11, October 25, November 8, November 22.
2. An end of term presentation, including a short written report. The presentations will be mostly done in pairs. The presentations will be scheduled during the last 8 lectures of the term. (Updated September 26).

The course grade is computed as: Homework: 60% (each assignment is equally weighted),

Presentations (oral and written components): 40%.

Policy on late or missed assignments. Accommodations for missed or incomplete assignments will be made on a case by case basis. Please contact the instructor as soon as possible if you anticipate missing an assignment. Each student is allowed to submit one homework assignment up to 3 days late without penalty (Added September 26). Outside of rare emergencies, other late assignments will not be accepted without prior arrangement.

Policy on collaboration and use of online tools:

- You are encouraged to discuss the homework problems with your classmates, but you must write up your solutions independently, and acknowledge any collaboration.
- Use of existing internet resources such as Wikipedia, Math Stack Exchange, etc. *is allowed*, but you must acknowledge any sources you use. Posting questions directly related to HW or final presentations on these sites is *not* allowed.
- Use of artificial intelligence tools *is allowed* for both the homework assignments and the final presentation. However, you must explicitly indicate how such tools were used in your submission (e.g. "used chatGPT to fix a latex error"). Be aware that even if a LLM may provide reasonably-looking full answers to HW problems, they are extremely likely to be wrong.

Syllabus Policies

General UBC and Mathematics Department policies can be found here.

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