

Your solution for each problem below should be understandable by an average student at the given grade level, concise but with all necessary details, and well-organized. At the beginning of the solution, briefly motivate the solution method before going into details, as you would if you were presenting the solution in front of a class. Include comments about the problem aimed at workshop leaders or teachers.

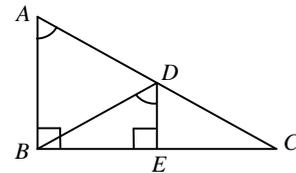
This exam is out of 100 points.

Grade 6–7 Level:

1. (10 points) In a bakery, several different types of dessert are sold to the customers. Each dessert has sponge cake, cheesecake, mousse, or pudding as its base. A customer must add exactly two different toppings chosen from strawberries, blueberries, chocolate sprinkles, and cookie wafers, but cannot choose both fruits. How many different types of dessert does the bakery sell (the order of the toppings doesn't matter)?
2. (15 points) There are 26 cards, each of which has a letter on one side and a number on the other side. You are told that the cards satisfy the rule: "if there is a vowel on one side, then there is an even number on the other side." You can see one side of each card. Eight cards have the numbers 22, 23, 31, 33, 44, 45, 47, and 48. The other 18 cards have the letters A, B, C, ..., R. What is the smallest number of cards you must turn over to check whether the rule is satisfied?

Grade 8–10 Level:

3. (15 points) In the diagram, triangles ABC and DEB are right-angle triangles and ABC is similar to triangle DEB . Also, the length of BC is $13\frac{1}{3}$ cm and the length of AB is 10 cm. Find the length of BD .



4. (15 points) A hotel owner is very superstitious and avoids room numbers that include the string "13." For example, he doesn't number any room 213 or 1395. The first room is numbered 101, and the other rooms are given increasing consecutive numbers, with the unlucky numbers containing 13 skipped. The last room has is numbered 2009. How many rooms are in the hotel?

Grade 11–12 Level:

5. (15 points) An ice-cream cone has height 15 cm, and the diameter of its circular opening is 10 cm. A single scoop of ice cream, in the shape of a sphere, is placed in the cone so that the centre of the scoop coincides with the centre of the circular opening of the cone. What is the radius of the scoop?
6. (15 points) A stool with a circular seat is pushed into the corner of a room so that the seat touches both walls. A marked point on the edge of the seat is measured to be 5 inches from one wall and 10 inches from the other. What are the possible diameters of the stool's seat?
7. (15 points) Find all solutions of the system of equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$.