Be sure that this examination has 3 pages.

The University of British Columbia

Final Examinations - April 23 2014

Mathematics 400

Juncheng Wei

Closed book examination | Time: $2\frac{1}{2}$ minutes

Special Instructions: No notes, books, or calculators are allowed.

Marks

[20] **1.** Consider the following first order PDE

$$u_t + u^3 u_x = 0, \quad t > 0, \quad -\infty < x < +\infty$$

with $u(x, 0) = 1$ when $0 < x < 1$ and $u(x, 0) = 0$ otherwise.

(i) (15) Find the solution with expansion fan and shock.

(ii) (5) Locate the time t_B when the expansion fan hits the shock. Find the solution when $t > t_B$.

Continued on page 2

[25] **2.** Consider the following wave equation

$$u_{tt} - 4u_{xx} = f(x,t), \quad 0 < x < +\infty, \ t > 0$$

$$u(x,0) = \phi(x), \ u_t(x,0) = \psi(x), \ 0 < x < +\infty$$

$$u(0,t) = h(t), \ t > 0$$

(i) (10) Find the general solution to the above wave equation when $f = 0, \phi = 0$ and $\psi = 0$

(ii) (10) Find the solution to the above wave equation with

$$f(x,t) = xt, \quad \phi(x) = 1, \quad \psi(x) = \sin x, \quad h(t) = e^t$$

(iii) (5) Use the energy method to show that the solution to the above wave equation is unique.

[25] **3.** Consider the following diffusion equation

$$u_t = u_{xx} + 2u_x + u, \quad 0 < x < 1, \quad t > 0$$

$$u(x,0) = \phi(x), \quad 0 < x < 1$$

$$u(0,t) = 0, \qquad 2u_x(1,t) - u(1,t) = 0, \ t > 0$$

(i) (20) Use the method of separation of variables to find the general solution.

(ii) (5) Under what condition on the initial condition ϕ is the solution obtained in (i) bounded? Justify your answer.

Continued on page 3

[10] **4.** Consider the following Laplace equation $u_{xx} + u_{yy} = 0$ in the disk of radius a defined by $D = \{(x, y) \mid x^2 + y^2 < a^2\}$, with $u(x, y) = 1 + x^2 + 3xy$ on the boundary of D: $x^2 + y^2 = a^2$.

(5) Without solving the problem explicitly, find the value of $u(0, 0)$, and the maximum and minimum values of u in D. (5) Justify your answer.

[20] **5.** Use the method of separation of variables to solve the following Laplace equation

$u_{xx} + u_{yy} = 0$ in $D = \{(x, y) \mid x^2 + y^2 > 4,\ x > 0,\ y > 0\}$,

$u_y(x, 0) = 0$ for $x > 0$, and $u(0, y) = 0$ for $y > 0$,

$u(x, y) = y^2$ on $x^2 + y^2 = 4, x > 0, y > 0$,

$u(x, y)$ is bounded.

[100] **Total Marks**

The End