THE UNIVERSITY OF BRITISH COLUMBIA

Sessional Examinations—April 2008

Mathematics 335

Time: $2\frac{1}{2}$ hours

This examination has 2 pages. Please check that you have a complete paper. A *simple* calculator may be used, but only to add, subtract, multiply, or divide. No other special aids (books, notes, instant messaging) are to be used.

A. Please do **3** of parts (i)–(v). Each part is worth 7 marks. If you attempt more than 3 parts, indicate clearly which 3 you want marked.

(i) Write a couple of substantial paragraphs about *Euclid.* Identify the time and place at which he was active, and describe in some detail his connections with the mathematics looked at in this course.

(ii) Write a couple of substantial paragraphs about *Simon Stevin.* Identify the time and place at which he was active, and describe in detail his contribution to the mathematics looked at in this course.

(iii) Write a couple of substantial paragraphs about prime numbers. Include a definition of prime number, and several additional facts and results.

(iv) Show that $\sqrt{2}$ is irrational.

(v) Give a proof of the Pythagorean Theorem.

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Please do **7** of the following 9 questions. Each is worth 10 marks. If you do more than 7 of the questions, indicate clearly which 7 you want marked. Solutions should be as detailed as time allows.

1. The number N will be called *good* if N is a 4-digit number, that is, a number between 1000 and 9999, and the decimal representation of N begins or ends with an odd digit. For example, 3004, 6777, and 3565 are good. How many good numbers are there? Give an explicit numerical answer.

2. Calculate in an efficient way the sum

$$7 + 10 + 13 + 16 + \cdots + 2002 + 2005 + 2008.$$

(Each term is 3 more than the previous one.)

3. Of the people in attendance at an international meeting, 220 can speak neither English nor French. Of the rest, 1500 can speak English, 600 can speak French, and 110 can speak French but cannot speak English. In total, how many people are at the meeting?

4. At TopHeavy™ Pizza, all pizzas have the same diameter, and there are 20 different toppings available. A 3-Top™ is a pizza with exactly 3 scoops of topping. One example of a 3-Top has 1 scoop each of anchovy, artichoke, and arugula. Other examples of a 3-Top have 1 scoop of almond and 2 of asparagus, or 3 scoops of peanut butter. How many different 3-Top pizzas are there? Find an explicit numerical answer.

5. Let $N = 2^{10}3^{11}5^{12}$. How many positive perfect cubes divide N? (An integer b is a perfect cube if there is an integer a such that $b = a^3$.)

6. Let $b = 88888$. Find the remainder when b^4 is divided by 4321. Even if your calculator has fancy features, pretend it is a simple hand-held "grocery store" calculator. Use no operator keys other than addition, subtraction, multiplication, and division. Describe in detail the steps in your calculation.

7. A certain 26-digit number N has the decimal representation

$$555555555555x555555555555y,$$

where x and y are digits, that is, integers in the interval from 0 to 9. Given that N is a multiple of 36, what are the possible pairs (x, y)?

8. Let a be the number whose base eight representation is 1357. So, to be formal, $a = (1357)_{\text{eight}}$. Find the base eight representation of $a + a + a$. (Use as "digits" 0, 1, 2, 3, 4, 5, 6, 7, with their usual meanings.)

9. Let A be the collection of all integers which have 7 digits in their decimal representation. (The smallest of these is 1000000, and the largest is 9999999.) How many of the numbers in A have exactly three 5's in their decimal representation? Please simplify.