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The University of British Columbia
Sessional Examinations - December 2014

Mathematics 320
Real Variables I

Closed book examination Time: $2\frac{1}{2}$ hours

Name ______________________ **Signature** ______________________

Student Number ______________________ **Instructor's Name** ______________________

Section Number ______________________

Special Instructions:

No books, notes, or calculators are allowed.

Rules Governing Formal Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

i. speaking or communicating with other examination candidates, unless otherwise authorized;

ii. purposely exposing written papers to the view of other examination candidates or imaging devices;

iii. purposely viewing the written papers of other examination candidates;

iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) — (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

7. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		10
2		15
3		15
4		15
5		15
6		15
7		15
Total		100

Marks

[10] **1.** Define

(a) supremum

(b) limit point of a set

(c) $\lim_{x \to p} f(x)$

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[15] **2.** Give an example of each of the following, together with a brief explanation of your example. If an example does not exist, explain why not.

(a) A sequence $\big(a_n\big)_{n\in\mathbb{N}}$ of real numbers, converging to 0, for which the partial sums $s_n = \sum_{k=1}^{n} a_k$ are bounded, but $\sum_{k=1}^{\infty} a_k$ diverges.

(b) Two subsets, A and B, of a metric space for which $\overline{A\cap B} \neq \overline{A}\cap\overline{B}$.

(c) A function $f : \mathbb{R} \to \mathbb{R}$ such that

$$\forall\varepsilon > 0\ \exists\delta > 0 \text{ s.t. } |f(x) - f(y)| < \varepsilon \implies |x - y| < \delta$$

and yet f is not continuous.

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[15] **3.** Let A and B be two nonempty sets of rational numbers that obey the following two conditions.

(i) If x is a rational number then x is in either A or B, but not both.

(ii) If $a \in A$ and $b \in B$ then $a < b$.

Prove that there is one and only one real number γ such that $a \le \gamma \le b$ for all $a \in A$ and $b \in B$.

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[15] **4.** Let $\left(a_n\right)_{n\in\mathbb{N}}$ be a sequence of real numbers with the property that there exists a real number $0 < r < 1$, and an integer N_0 such that for all $n \geq N_0$, $|a_n - a_{n-1}| \leq r|a_{n-1} - a_{n-2}|$. Prove that $\left(a_n\right)_{n\in\mathbb{N}}$ converges.

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[15] **5.** Denote by E the set of all real numbers x for which the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{n^2 4^n}$ converges absolutely. For each $x \in E$, set $f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n^2 4^n}$.

(a) Determine E.

(b) Prove that $f(x)$ is continuous on E.

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[15] **6.** Let $(\mathcal{M}, d)$ be a metric space. The subset $S \subset \mathcal{M}$ is said to be path connected if for each pair p, q of points in S there is a continuous function $f : [0, 1] \to S$ such that $f(0) = p$ and $f(1) = q$. Prove that any path connected subset of $\mathcal{M}$ is connected.

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[15] **7.** Let (X, d_X) and (Y, d_Y) be metric spaces with X compact. Let $f : X \to Y$ be a continuous function. Prove that for each $\varepsilon > 0$, there is a $\delta > 0$ such that $d_Y\big(f(a), f(b)\big) < \varepsilon$ for all $a, b \in X$ obeying $d_X(a, b) < \delta$.

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The End