Math 309
Final Exam
April 19, 2008
Duration: 150 minutes

Name: _____ Student Number: _____

Do not open this test until instructed to do so! This exam should have 11 pages, including this cover sheet. No books or notes are allowed. You may use a basic (nonscientific) calculator. Turn off any call phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam.

UBC Rules governing examinations:

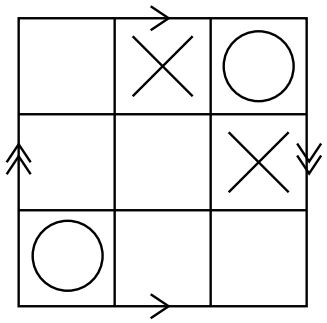
(1) Each candidate should be prepared to produce his/her library/AMS card upon request.
(2) No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.
(3) Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:
a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
b) Speaking or communicating with other candidates.
c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.
(4) Smoking is not permitted during examinations.

Problem	Out of	Points
1	6	
2	4	
3	8	
4	10	
5	6	
6	6	
7	8	
8	8	
9	9	
10	10	
Total	75	

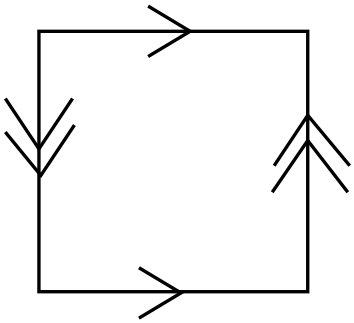
1. Short Answer

No justification needed. Clearly indicate your final answer.

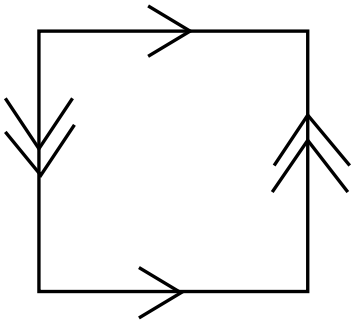
(6 pts) **1** (a) What is the best move for X in the following game of Klein bottle tic-tac-toe?



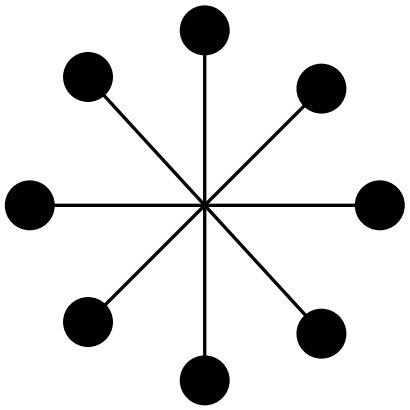
(b) Indicate where to cut the Klein bottle to wind up with two annuli (regular tubes).

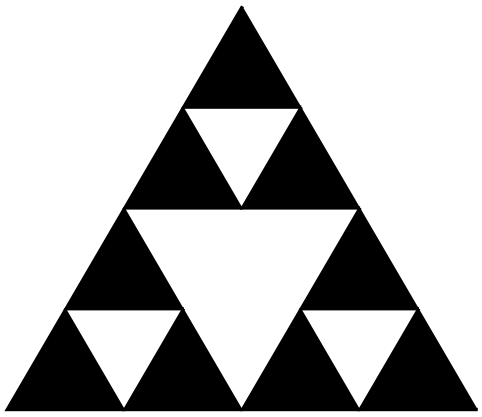


(c) Indicate where to cut the Klein bottle to wind up with two Möbius bands.



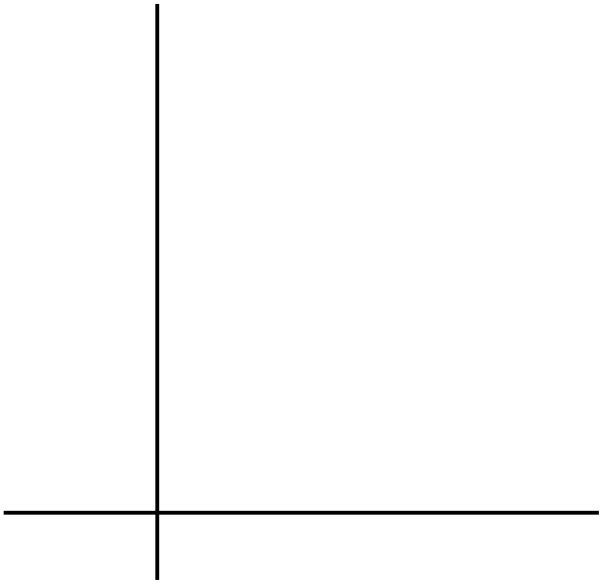
(4 pts) **2** For each of the following figures, state what symmetry type tthe figure is, and write the result of the combination of symmetries $r^{10}mr^2mr^3m$ in standard form.

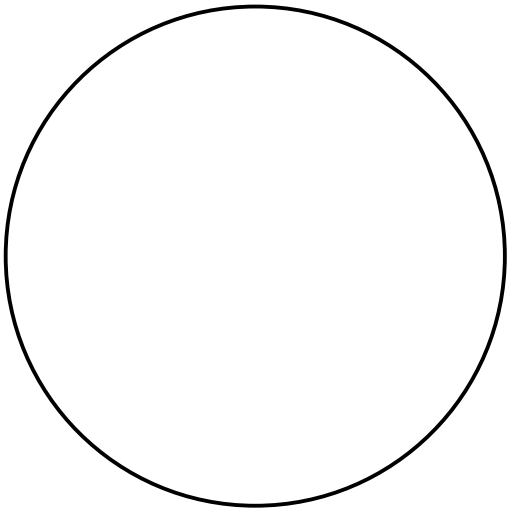




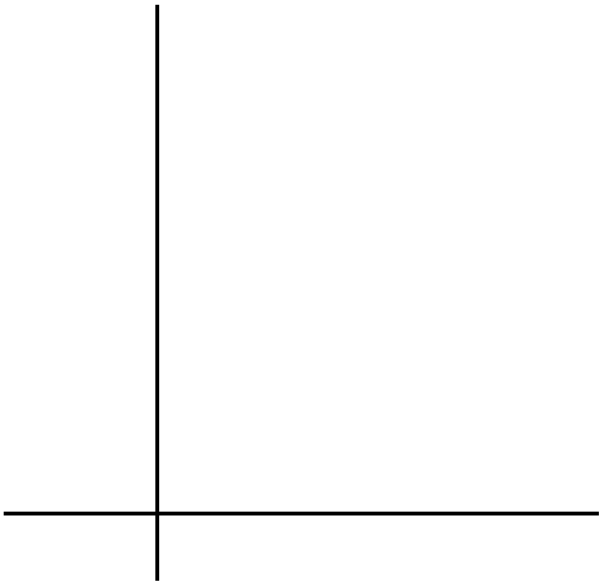
(8 pts) **3** Draw reasonably accurate sketches in each model of the hyperbolic plane to illustrate the following in the hyperbolic plane. Be sure to label your sketches clearly.

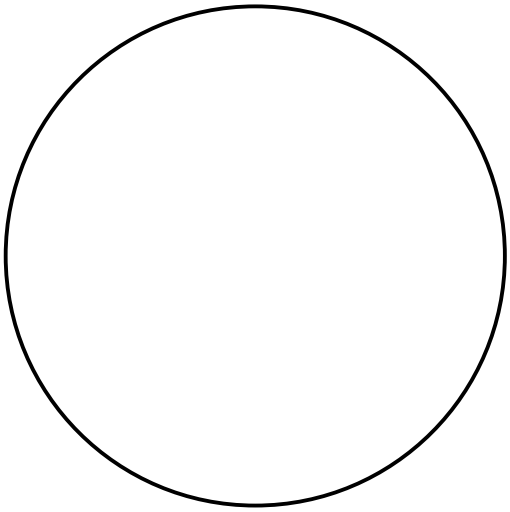
(a) The existence of multiple lines parallel to a given line ℓ through a given point P:
upper half-plane model Poincare disc model





(b) The defect of a hyperbolic triangle is proportional to its area:
upper half-plane model Poincare disc model





(10 pts) **4** For each potential combination of symmetries of strip patterns, either draw an example of a strip pattern with that symmetry type or state briefly why it is impossible.

(a) translation and rotation by $\frac{1}{2}$ (and no other symmetries)

(b) translation and reflection in a horizontal mirror line (and no other symmetries)

(c) translation and glide reflection (and no other symmetries)

(d) translation and reflections in both horizontal and vertical mirror lines (and no other symmetries)

(6 pts) **5** In class we talked about building higher dimensional cubes. We can also do higher dimensinal triangles. A 1-d triangle is a line segment. A 2-d triangle is the usual triangle, and a 3-d triangle is a tetrahedron. To make each triangle from the one before, we take the lower dimensional triangle and add a single point beside it in a new direction ("above"). Then we draw new edges from the vertices of the original to our new point.

(a) Sketch a 4-dimensional triangle.

(b) Fill in the following chart.

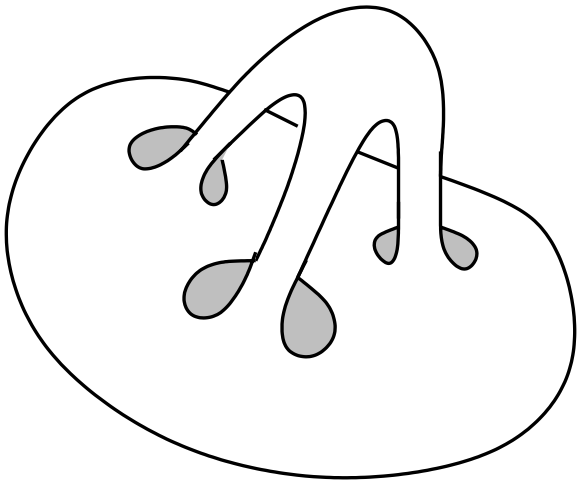
Dimension of triangle	# vertices v	# edges e	# faces f	# 3-d solids
1				
2				
3				
4				
5				

2. Long Answer

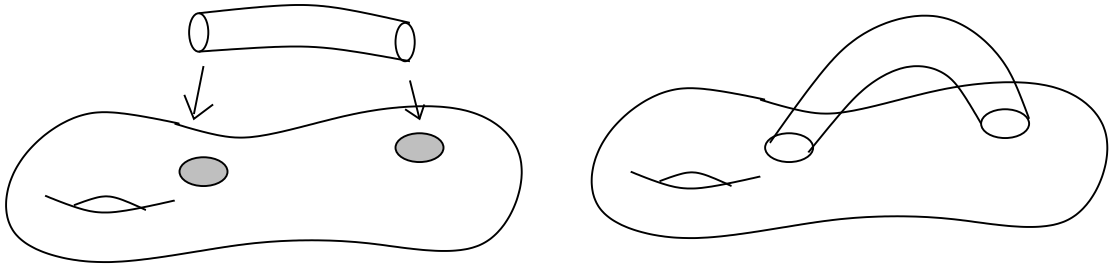
Justify all answers, show all work and **explain your reasoning carefully**. You will be graded on the clarity of your explanations as well as the correctness of your answers.

(6 pts) **6** Which space is more curved, a direct sum of projective planes $P\sharp P\sharp P$ with an area of 6, or a double torus $T\sharp T$ with an area of 9?

(8 pts) **7** Identify the following surface.



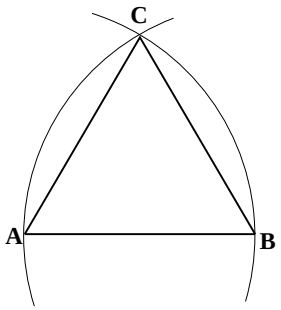
(8 pts) **8** We can take any surface and add a handle to it, by cutting out two holes and gluing the handle in as shown:



(a) What is the Euler characteristic of the new surface $S + h$ (with the handle) compared to that of the original surface S?

(b) What homogeneous geometry can be given to a Klein bottle with a handle added?

(9 pts) **9** The first proof from the axioms given in Euclid's elements is the construction of an equilateral triangle on a given base.
Given a line segment AB, there exists a circle centred at A with radius AB by Postulate 2. Similarly, there exists a circle centred at B with radius BA. Let C be the intersection of these circles; then $AB = AC$ (since all radii of circles are equal), and $BA = BC$ (similarly) and so $AB = AC = BC$ and $\triangle ABC$ is equilateral.



Comment on whether this proof is valid in spherical, hyperbolic, and taxicab geometry.

(10 pts) **10** (a) Briefly describe Klein's Erlanger program for geometry.

(b) Explain why 'slope' of a line is a valid geometric measurement in translational geometry, but not in the usual Euclidean geometry.

(c) Give an Erlanger-style proof in Euclidean geometry of the following: If a triangle is inscribed in a circle, the side lengths satisfy the Pythagorean formula $a^2 + b^2 = c^2$. (This implies that the angle is a right angle, but you don't need to prove this here.)