

University of British Columbia
Math 305, Section 101 (Froese)
Final Exam, December 2014

Name (print): _____

Student ID Number: _____ Signature: _____

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Question	Points	Score
1	7	
2	8	
3	7	
4	7	
5	8	
6	8	
7	10	
8	9	
9	10	
10	8	
11	8	
12	10	
Total:	100	

Additional Instructions:

- No notes, books or calculators are allowed.
- Read the questions carefully and make sure you provide all the information that is asked for in the question.
- Show all your work. Correct answers without explanation or accompanying work could receive no credit.
- Answer the questions in the space provided. Continue on the back of the page if necessary.

1. (7 points) What is the image of the unit circle $\{z : |z| = 1\}$ under the mapping $f(z) = z - 1/z$?

2. Let $f(z) = |z|^2$.

- (a) (3 points) For which $z = x + iy$ do the real and imaginary parts of $f(x + iy)$ have continuous partial derivatives with respect to x and y ?
- (b) (3 points) For which z is $f(z)$ complex differentiable?
- (c) (2 points) For which z is $f(z)$ analytic?

3. (7 points) For which z is it true that $\text{Log}(1/z) = -\text{Log}(z)$? (Here Log denotes the principal branch of the logarithm.)

4. (7 points) What are all the values of the multivalued expression i^i ?

5. Let $f(z)$ be an analytic function in some connected domain D (i.e., any two points in D can be connected with a path).

(a) (4 points) Does it follow that f has an antiderivative in D ? Give a reason.

(b) (4 points) If $f'(z) = 0$ everywhere in D , does it follow that $f(z)$ is constant? Give a reason.

6. (8 points) Find $\max_{z:|z|\leq 1} |z^n + i|$. Indicate what facts about analytic functions you are using in your calculation.

7. (10 points) Compute the Laurent expansion for $\frac{z^n}{z^2 - 1}$ for $n \in \mathbb{Z}$ valid in the region $|z| > 1$ and use it to compute $\oint_{|z|=5} \frac{z^n}{z^2 - 1} dz$, where the circle is traversed once in the counterclockwise direction.

8. (9 points) Explain why $\cos(1/\sqrt{z})$ is analytic in the punctured plane $\mathbb{C}\setminus\{0\}$ and compute

$$\oint_{|z|=1} \cos(1/\sqrt{z})dz,$$

where the circle is traversed once in the counterclockwise direction. Here \sqrt{z} is any branch of the square root.

9. (10 points) The following integral can be computed as

$$I = \int_0^\pi \frac{\cos(2\theta)}{2 + \cos(\theta)} d\theta = 2\pi i \left(\operatorname{Res} [f(z), z_0] + \operatorname{Res} [f(z), z_1] \right).$$

What are $f(z)$, z_0 and z_1 ?

10. (8 points) Compute

$$I = \int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 4)^2} dx.$$

11. (8 points) Compute

$$I = \int_{-\infty}^{\infty} \frac{\cos(x)}{x + 2i} dx.$$

12. (10 points) Compute

$$I = \int_0^{\infty} \frac{\ln(x)}{x^2 + 4} dx.$$

Include the error estimates needed to justify your calculation.