Be sure this exam has 9 pages including the cover

The University of British Columbia

MATH 305

Final Exam – Dec 6, 2013

Family Name _____ **Given Name** _____

Student Number _____ **Signature** _____

This exam consists of **3** parts. No notes nor calculators. Note the number of marks for each question. Use your time wisely. **Time**: $2\frac{1}{2}$ hours

Problem	max score	score
1.	8	
2.	28	
3.	14	
total	50	

(8 points) 1. Answer the following multiple choice questions. Check your answer very carefully. Your answer will be marked right or wrong (work will not be considered for this problem).

(2 points) (a) Let $f(z) = \frac{1}{z^2(1-z)^3}$. Then the residue of f at $z = 1$, denoted as $\text{Res}(f; 1)$ is

A. 6
B. -6
C. 3
D. -3
E. None of the above.

(a) _____

(2 points) (b) Let $f(z) = 1/(z^2 + 1)$. Let Γ be the positively oriented eclipse $\{z = x + iy : \ x^2 + \frac{(y-2)^2}{4} = 1\}$. Then $\int_\Gamma f(z)dz =$

A. 0
B. π
C. $-\pi$
D. $\pi/2$
E. None of the above.

(b) _____

(2 points) (c) Which of the following is/are true? (There may be more than one choices!)

A. The function $f(z) = \sin z/\cos z$ has only isolated singularities which are simple poles
B. The function $f(z) = \frac{\cos z}{z^2} - \frac{1}{z^2}$ is bounded in the disk $\{z : |z| \leq 1\}$
C. There does not exist a bounded function which is analytic in the region $\{z : |z| > 1\}$
D. None of the above hold true.

(c) _____

(2 points) (d) $\sum_{k=1}^{\infty} \frac{1}{k^2+4} =$ (hint: use Residue Theory)

A. $\pi \frac{\cosh(2\pi)}{\sinh(2\pi)}$
B. $\pi \frac{\sinh(2\pi)}{\cosh(2\pi)}$
C. $\frac{\pi}{4} \frac{\sinh(2\pi)}{\cosh(2\pi)}$
D. $\frac{\pi}{4} \frac{\cosh(2\pi)}{\sinh(2\pi)}$
E. None of the above

(d) _____

(28 points) 2. Compute the following integrals and simplify your answer to the best possible. Show in detail how you arrive at your answer (work will be considered for this problem, DO NOT omit steps!).

(8 points) (a) Find the Laurent series for $f(z) = \frac{1}{(z-1)^2(z-3)}$ valid for (A) $0 < |z-1| < 2$ and (B) $0 < |z-3| < 2$. You need to write out the explicit expression for the first four terms.

(8 points) (b) By using the Residue Theory, compute the integral $\int_{-\infty}^{\infty} \frac{x \cos x}{x^2-3x+2} dx$

(5 points) (c) By using the Residue Theory, compute the integral

$$I = \int_0^\infty \frac{x}{x^3+1} dx$$

(c) _____

(7 points) (d) The value of the integral $\int_0^{\pi} \frac{\cos(x-i)}{\sin(x+i)} dx$ is

(d) _____

(14 points) 3. By using the Residue Theory, compute the following integrals. You must explain clearly which theorems you are using and justify all your estimates.

(7 points) (a) Compute

$$I = P.V. \int_0^\infty \frac{(1-\cos x)^2}{x^4} dx.$$

(7 points) (b) Let $0 < \lambda < 1$. Compute

$$I = \int_0^\infty \frac{1}{x^\lambda (x+1)(x+4)} dx.$$

This is an extra page for answers.