

- 1) A poker hand consist of 5 cards out of a deck of 52.
 - a) What is the probability of a *Full House*, that is, three cards of one denomination and two cards of a second denomination. [5]
 - b) What is the probability of *One Pair*, that is, two cards from one denomination, with the third, fourth and fifth card from a second, third, and fourth denomination, respectively: for example two 8's, a king, a queen and a 4. [5]

2) Mrs Smith travels from Berlin to Vancouver via Frankfurt and Calgary. There is a 5% chance that her luggage is left behind in Berlin. If it is not left behind in Berlin, there is a 7% chance that it is left behind in Frankfurt. If it not left behind in Berlin or Frankfurt, there is a 10% chance that it is left behind in Calgary.

- a) What is the probability that she can claim her luggage in Vancouver? [5]
- b) Suppose that her luggage was left behind. What is the probability that it was left behind in Calgary? [5]

3) Past experience shows that 30% of the customers entering Harry's Clothing Store will make a purchase. Of the customers who make a purchase, 85% use credit cards. Let X be the number of the next 6 customers who enter the store, make a purchase, and use a credit card. Find the expected value and the variance of X . [10]

4) On average, two cars in a hundred brake due a particular mechanical failure (independently of each other) which can only be detected in a complex check. Use the Poisson approximation to the Binomial distribution to estimate how many cars should be tested in order that the probability is at least 0.95 that at least one of the cars will have the defect. [10]

5) Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} e^{-y} & \text{if } 0 \leq x \leq y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

- a) Find $\mathbb{P}(Y \geq 2X)$ and $\mathbb{P}(Y \leq 2X)$. [10]
- b) Determine the covariance of X and Y . [10]

6) Let X denote the random lifetime of a TV (in years) and assume that the density takes the form

$$f(x) = \begin{cases} 2e^{-x/2} & \text{if } 0 \leq x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

- a) Calculate the probability that exactly 3 out of 4 TVs last for more than 4 years. [5]
- b) Use both the transformation formula AND the method of the distribution function to find the density of $Y = \sqrt{X}$. [5]

7) An exam has 100 true/false questions. Suppose a student knows the correct answers to 36 questions, but chooses true/false with equal probability for the remaining questions. Use the normal approximation to the binomial to estimate the probability that the student gets 72 questions or more correct. [10]

8) Mr Smith invests \$100.000 equally in four uncorrelated assets whose annual returns r_1, \dots, r_4 are normally distributed with respective means 8%, 10%, 12%, and 14% and standard deviations 6%, 7%, 5%, and 12%, respectively. What is the probability that, after one year, Mr Smith's net profit from this investment is at least \$15.000? [10]

9) Let X be a continuous random variable with density

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- a) Calculate the mean μ and the variance σ^2 of X . [5]
- b) Use Chebyshev's inequality to estimate the probability that X deviates from its mean by more than two standard deviations. [5]