

The University of British Columbia

Final Examination - April 25, 2015

Mathematics 301/201

Closed book examination

Time: 2.5 hours

Last Name _____ First _____ Signature _____

Student Number _____

Special Instructions:

Calculators are allowed, but no books or notes.

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)-(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

[10]. 1. Prove that all five zeros of the polynomial

$$g(x) = z^5 + 3z + 1 \text{ lie in the disk } |z| < 2.$$

[10] 2. Suppose we have two circles $x^2 + y^2 = 4$ and a smaller circle $(x - 1)^2 + y^2 = 1$. Suppose $T_{xx} + T_{yy} = 0$ between circles. If $T = 1$ on the smaller circle and $T = 5$ on the larger circle, then find the value of the function T at each point between the circles.

[10] 3.

(a) Map the circle $|z| < 3$ to the lower half plane with a bilinear map.

(b) Find a map to take the infinite strip $0 \leq \operatorname{Re} z \leq a, \operatorname{Im} z \geq 0$ to the upper half of the w -plane.

[10] 4. Evaluate the integral

$$\int_{-\infty}^{+\infty} \frac{x \cos(\pi x) dx}{x^2 + 2x + 5}$$

[10] 5. Use the Fourier transform to solve the diffusion equation:

$$u_t = D u_{xx} \quad -\infty < x < \infty \quad t > 0$$

$$u(x, 0) = f(x) \quad u(x, t) \rightarrow 0 \text{ as } |x| \rightarrow \infty$$

[10] 6. Evaluate the following integrals using residues

(a) $I = \int_C \frac{dx}{z^2(z+2)}$ where C is $|z| = 3$

(b) $I = \int_C \tan(z)dz$ where C is $|z| = 4$

[10] 7. Consider the following initial value problem for $y(t)$:

$$y'''' + ky''' + y'' + y' = e^{-t} \quad t \geq 0, \quad k \text{ real with } k \geq 0$$

$$y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1, \quad y'''(0) = 4.$$

Calculate the Laplace transform of $y(t)$, denoted by $Y(s)$, in the form $Y(s) = P(s)/Q(s)$ where P and Q are polynomials.

[10] 8. Consider

$$y' + y = f(t) \text{ with } f(t+1) = f(t) \text{ with initial value } y(0) = a.$$

Show how to find $y(0) = a$ so that the solution is periodic with $y(t+1) = y(t)$.

[10] 9.

(a) Define z^α for z and α complex numbers

(b) Find all values of $2^{\pi i}$.

(c) Find all values of $(1+i)^{(1+i)}$.

[10] 10.

(a) Define $\log z$ for z a complex number.

(b) Evaluate $\log(1 - i)$.

(c) Solve the equation

$$\text{Log}(z^2 - 1) = i\pi/2.$$