

The University of British Columbia
Math 301 — Applied Complex Analysis
2014, April 30
Omer Angel

Surname: _____ First Name: _____ Student id. _____

Instructions

- **Explain** your reasoning thoroughly, and **justify** all answers (even if the question does not specifically say so).
- Calculators, but no other aides are permitted.
- If you need more space, additional paper is available. Always **note** which questions are solved in a separate booklet, or they may not be graded.
- Duration: **150** minutes.

Good luck, and enjoy the break.

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Question	Points	Score
1	10	
2	15	
3	20	
4	10	
5	15	
6	15	
7	15	
Total:	100	

(10) 1. Compute $\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2 + \pi^2}$. Justify all steps.

$f(x)$	$\hat{f}(k)$	$f(t)$	$F(s)$
$\frac{1}{1+x^2}$	$\pi e^{- k }$	e^{-at}	$\frac{1}{s+a}$
$e^{- x }$	$\frac{2}{1+k^2}$	$\sin(at)$	$\frac{a}{s^2+a^2}$
$e^{-x^2/(2\sigma^2)}$	$\sqrt{2\pi}\sigma e^{-\sigma^2 k^2/2}$	$\cos(at)$	$\frac{s}{s^2+a^2}$

(15) 2. Compute $\int_0^\infty \frac{x^{-2/3}}{x+3}$. State which branch and the contour you are using. Justify all steps.

- (20) 3. Consider the branch of $(z^3 - 2z^2)^{1/3}$ given by $f(z) = \exp(\frac{2}{3} \text{Log } z + \frac{1}{3} \text{Log}(z - 2))$, where $\text{Log } z$ is the principle branch of $\log z$.
- a. Where are the branch points of $(z^3 - 2z^2)^{1/3}$? Justify your claim, do not forget a possible branch point at ∞ . Where are the branch cuts of f ?

b. Express f using the range of angles method.

c. Find the limit of $f(z)$ as z approaches 1 from above and from below.

d. Find the limit of $f(z)$ as z approaches -1 from above and from below.

- (10) 4. If $f(z) = \frac{P(z)}{Q(z)}$ is rational with $\deg(Q) \geq 2 + \deg(P)$, prove that the sum of the residue of f at its poles is 0.

- (15) 5. a. Map the region $\{|z - 2| > 1\} \cap \{|z + 2| > 1\}$ to an annulus $\{a < |z| < b\}$ for some a, b .
- b. Solve the Laplace equation $\Delta\varphi \equiv 0$ in that region with boundary conditions $\varphi = -1$ on the left circle and $\varphi = 1$ on the right circle. (Hint: $\log |z|$ is harmonic except at 0.)

(15) 6. Let $f(x) = \begin{cases} xe^{-3x} & x > 0 \\ 0 & x < 0 \end{cases}$.

a. Compute the Fourier transform of f .

b. Compute the Laplace transform of f .

- (15) 7. For a fixed number a , consider the solution of $y''(t) + ay'(t) + y(t) = \sin(3t)$ with $y(0) = 1$ and $y'(0) = 2$.
- Find the Laplace transform $Y(s)$.

b. Determine for which values of a the solution is bounded in t .