The University of British Columbia

Final Examination - December 11, 2012

Mathematics 265

Time: 2.5 hours

Last Name ____________________ **First** ______________ **Signature** ____________________

Student Number ____________________ **Section** ______________

Special Instructions:

- Books, notes, cellphones and calculators are **not** allowed.
- Use the reverse side of each page if you need extra space.
- Show your work. A correct answer without intermediate steps will receive no credit.
- This exam has 12 pages, including this cover and a table of Laplace transforms.
- You may detach the table of Laplace transforms when the exam starts.
  You may **not** write or mark the table of Laplace transforms, for any reason.

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  (a) speaking or communicating with other candidates, unless otherwise authorized;
  (b) purposely exposing written papers to the view of other candidates or imaging devices;
  (c) purposely viewing the written papers of other candidates;
  (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		15
2		5
3		15
4		15
5		10
6		10
7		10
8		10
9		10
Total		100

Problem 1 (15 Marks)

Solve $y' + y = 1$ for $y(t)$, subject to $y(0) = 1$.

$y(t) =$

Solve $t\frac{dy}{dt} - y = 2t^2 y$ for $y(t)$, subject to $y(1) = 1$.

$y(t) =$

Problem 2 (5 Marks)

Suppose that a bacterial population grows exponentially. At 11am there are 100 bacteria and at 1pm there are 400 bacteria. When was only one bacteria present?

Your answer will include logarithm functions.

Problem 3

(15 Marks)

(a) Solve the following problem, assuming $\omega \neq 4$:

$$\begin{cases} y'' + 16y = \cos(\omega t) \\ y(0) = y'(0) = 0 \end{cases}$$

$y(t) =$

Problem 3 (continued)

(b) Draw a graph of your solution to part (a) when $\omega = 5$.
Show at least one full period.

Hint: use the identity, $\cos(A) - \cos(B) = 2\sin\left(\frac{B-A}{2}\right)\sin\left(\frac{B+A}{2}\right)$.

Problem 4 (15 Marks)

Find the inverse Laplace transform of $F(s) = \frac{3s+2}{s^2+6s+9}$.

$f(t) =$

Find the convolution of $f(t) = t$ and $g(t) = e^{-t}$.

$(f * g)(t) =$

Problem 5 (10 Marks)

Use Laplace transform to solve the following problem.
Draw a clear graph of the solution, showing period and amplitude.

$$\begin{cases} y''(t) + \pi^2 y(t) = -\pi\delta(t-2) \\ y(0) = 0 \quad y'(0) = \pi \end{cases}$$

$y(t) =$

Problem 6 (10 Marks)

Find the solution of the differential equation system:

$$y' = \begin{pmatrix} 7 & 3 \\ 5 & -7 \end{pmatrix} y \qquad\qquad y(0) = \begin{pmatrix} 4 \\ -4 \end{pmatrix}.$$

$y(t) =$

Problem 7 (10 Marks)

Find the general solution:

$$x' = \begin{pmatrix} 2 & -4 \\ 1 & 2 \end{pmatrix} x$$

Your final answer should not be complex.

$x(t) =$

Problem 8 (10 Marks)

Find the general solution:

$$x' = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} x$$

Your final answer should not be complex.

$x(t) =$

Problem 9 (10 Marks)

(a) The second-order linear homogeneous equation

$$t^2 y'' + 6ty' + 4y = 0$$

has two linearly independent solutions of the form t^n. Find them.

$y_1 =$	$y_2 =$

(b) Find the solution of the forced equation

$$t^2 y'' + 6ty' + 4y = t \qquad y(1) = y'(1) = 0.$$

$y(t) =$

$f(t)$	$F(s) = \mathcal{L}[f(t)]$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$\delta(t-c)$	e^{-cs}
$e^{ct}f(t)$	$F(s-c)$
$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$