

- [12] 1. (a) Find the *scalar* projection of the vector $\langle 1, \pi, 2\pi \rangle$ along the vector $\langle 2, 6, 3 \rangle$.
- (b) Find the *vector* projection of $\langle 1, \pi, 2\pi \rangle$ along the vector $\langle 2, 6, 3 \rangle$.
- (c) Find the distance of the point $(2, \pi + 14, 2\pi - 27)$ from the plane given by the equation $2(x - 1) + 6(y - 14) + 3(z + 27) = 0$.

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- [12] 2. In an experiment, the light intensity is an unknown function $I(x, y, z)$ (it depends on the point in space). Its gradient at the point $(1, 2, 3)$ is given:

$$\nabla I|_{(1,2,3)} = \langle 0.1, -0.2, 0.5 \rangle$$

(we ignore the units).

- (a) Find the directional derivative $D_u I$ in the direction of the vector $\langle 1, 4, 12 \rangle$.

- (b) A small light sensor is moving through this space, with the position function

$$\mathbf{r}(t) = \langle t, 2t^2, 3t^4 \rangle.$$

Find the velocity of the sensor at the time $t = 1$.

- (c) Find $\frac{dI}{dt}|_{t=1}$ – the rate of change of intensity that the sensor (with the position function from part (b)) registers at the time $t = 1$.

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- [10] **3.** Find the absolute minimum of the function

$$f(x, y) = 2x^2 + y^2 + xy - 5x - 3y + 4$$

on the closed triangle bounded by the x -axis, the line $x = 2$, and the line $y = 2x$. (For full credit, it is sufficient to list *all* the points where you have to evaluate the function to find the minimum; you do not have to evaluate the function at all these points).

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- [10] 4. The double integral of some function over a domain D is represented by the iterated integral as follows:

$$\iint_D f(x, y) dA = \int_0^1 \int_{\sqrt{x}}^{2\sqrt{x}} f(x, y) dy dx.$$

- (a) Sketch the domain D .
- (b) Change the order of integration, so that in the result, integration with respect to y is on the outside.

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- [8] 5. A wire that occupies the segment in space between the points $(0, 1, 1)$ and $(2, 1, 4)$ has the density given by the formula

$$\rho(x, y, z) = ye^x + z,$$

(we ignore the units). Find the total mass of the wire.

[15] **6.** The vector field \mathbf{F} is defined on the plane by the formula

$$\mathbf{F}(x, y) = \langle y^2 \cos(xy^2), 2xy \cos(xy^2) + 3y^2 \rangle.$$

(a) Is \mathbf{F} conservative? Explain why or why not.

(b) Find a potential function for \mathbf{F} if it exists, or explain why it does not exist.

(c) Find the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ in any way you like, where C is the curve parametrized by the function $\mathbf{r}(t) = \langle t^2, t^3 \rangle$, $0 \leq t \leq 1$. (Here \mathbf{F} is the same vector field as in parts (a), and (b)).

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[15] 7. The vector field \mathbf{F} is defined on the whole space by the formula

$$\mathbf{F}(x, y, z) = \langle y^3 + e^{4x^2}, -x^3 - \sin(y), 3z^4 \rangle.$$

(a) Find $\mathbf{curl}(\mathbf{F})$.

(b) Evaluate (by direct computation) the flux integral

$$\iint_S \mathbf{curl}(\mathbf{F}) \cdot d\mathbf{S},$$

where S is the surface defined by $x^2 + y^2 \leq 1$, $z = 0$, (the unit disc in the xy -plane), oriented upward.

(This question is continued on the next page).

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- (c) Now let \mathbf{F} be the same vector field as in parts (a) and (b), and let M be the hemisphere $x^2 + y^2 + z^2 = 1$, $z \leq 0$, oriented downward. Evaluate

$$\iint_M \mathbf{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

in any way you like.

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- [18] **8.** (a) Let E be the solid inside the sphere of radius 2 centred at the origin, and above the cone defined by the equation $z^2 = \frac{1}{3}(x^2 + y^2)$. Using spherical coordinates, evaluate

$$\iiint_E z \, dV.$$

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- (b) Let \mathbf{F} be the vector field defined by the formula

$$\mathbf{F}(x, y, z) = \langle xz + y^2, x^2z^3, z^2 - y \rangle.$$

Compute $\mathbf{div}(\mathbf{F})$.

- (c) Let S be the closed surface that encloses the solid E from part (a) (so that S consists of the part of the cone that lies inside the sphere, and the spherical cap), oriented outward. Compute, in any way you like, the flux integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

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The University of British Columbia
Sessional Examinations - December 2010

Mathematics 263
Multivariable and Vector Calculus

Closed book examination

Time: 2.5 hours

Surname(s): _____ Given Name(s): _____

Student Number: _____ Instructor's Name: _____

Signature: _____ Section Number: _____

Special Instructions:

No books, notes, cell-phones or calculators are allowed, except for one letter-size formula sheet (you can use both sides). You must show all your work (i.e., intermediate steps) for full credit. If you need more space, use the back of the previous page, or ask for a booklet. You must turn in all the booklets you use.

Rules governing examinations

1. Each candidate should be prepared to produce, upon request, a UBCcard for identification.
2. No candidate shall be permitted to enter the examination room after the expiration of one-half hour, or to leave during the first half hour of the examination.
3. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices.The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.
7. Smoking is not permitted during examinations.

1		12
2		12
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4		10
5		8
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Total		100