

The University of British Columbia
Final Examination – April 24, 2013
Math 256

Closed Book Examination Time: **20 minutes**

Last Name _____ **First** _____ **Total Points** ____/15

Signature _____ **Student Number** _____

Special Instructions:

No books, notes, or calculators are allowed. **Show all your work:** little or no credit will be given for an answer without the correct accompanying work or explanation. If you need more space, use the back of the previous page.

Rules governing examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - i. speaking or communicating with other examination candidates, unless otherwise authorized;
 - ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
 - iii. purposely viewing the written papers of other examination candidates;
 - iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

MATH 256
Final Exam, Closed Book Section
April 24, 2013

Question 1. (2pts.)

Here is a set of two first order equations. Write it as an equivalent second order equation for a single unknown function. *Call the unknown function $y(t)$.* Your answer must be in the form of a single equation for $y(t)$.

$$dx_1/dt = x_2$$

$$dx_2/dt = -g x_2 - x_1$$

Question 2. (5 pts.)

Here is a matrix, **A**:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

(i) Find the eigenvalues and corresponding eigenvectors of **A**.

(ii) In the context of a system of ODEs, $dx/dt = \mathbf{A} \mathbf{x}$, does this system describe a saddle, a node, a center, or a spiral. Explain your answer.

Question 3. (6 pts.)

Here are three systems of ODEs. State whether each one of them is *autonomous* or *non-autonomous*. In addition, state whether they can be put into *matrix/vector form*. If 'yes', give the coefficient matrix in each case, and if 'no', explain why not.

(i) prey-predator with time dependent birth rate for the prey

$$dx/dt = a(t)x - xy;$$

$$dy/dt = -y + xy.$$

(ii) harmonic oscillator with an applied force $F(t)$ and a time-dependent spring constant

$$dx/dt = y;$$

$$dy/dt = -k(t)x - F(t).$$

(iii) competing species

$$dx/dt = x - x^2 - xy;$$

$$dy/dt = y - 2y^2 - 3xy.$$

Problem 4. (2 pts.)

Consider the following second order ODE.

$$d^2y/dt^2 + y dy/dt + \sin(t) = 0$$

Write this as a system of first order ODEs. Call the new variables $x_1(t)$ and $x_2(t)$.