The University of British Columbia

Final Examination - December, 2011

Mathematics 253

Closed book examination Time: 2.5 hours

Last Name: ____________________, **First:** _____________ **Signature** ____________________

Student Number ____________________

Special Instructions:

No books, notes or calculators are allowed.

Include explanations and simplify answers to obtain full credit.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		11
2		13
3		12
4		12
5		12
6		14
7		12
8		14
Total		100

[11] **1.**

A surface is given implicitly by

$$x^2 + y^2 - z^2 + 2z = 0.$$

(a) Sketch several level curves $z =$ constant
(b) Draw a rough sketch of the surface.
(c) Find the equation of the tangent plane to the surface at the point $x = 2,\ y = 2,\ z = 4.$

[13] **2**. A gas is known to satisfy the law

$$pV = T - \frac{4p}{T^2},$$

where p is the pressure, V is the volume and T is the temperature.

(a) Treating p and V as independent variables, find expressions for $\frac{\partial T}{\partial p}$ and $\frac{\partial T}{\partial V}$ in terms of p, V and T.

(b) Measurements of the pressure and the volume give $p = 1 \pm .001$ and $V = 1 \pm .002$. Find the approximate maximum percentage error made in the value of $T = 2$ given by the formula when $p = 1$ and $V = 1$.

[12] **3**. An ant is located at the point $x = 1, y = 1$ on a surface whose temperature is

$$T = f(x, y) = x^3 + yx.$$

(a) In what direction in the xy plane should the ant crawl so that its temperature T is *decreasing* the fastest?
(b) Suppose that the ant starts to crawl from $(1, 1)$ in the direction toward the point $(0, 2)$. At what rate is it's temperature changing if it's speed is 20?
(c) What is the cosine of the angle between the direction of maximum rate of *increase* in temperature and the positive y axis at the point $(1, 1)$?

[12] **4.** For

$$f(x, y) = x^3 - y^3 - 2xy + 6.$$

(a) Find the locations of all critical points of $f(x, y)$.
(b) Identify the critical points as local max/min and saddle points.

[12] **5**. Find the points on the ellipsoid $x^2 + xy + y^2 + yz + z^2 = 24$ which are farthest from the xy-plane.

[14] **6**. Consider the integral over the region R in the xy plane:

$$I = \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\, dxdy + \int_1^4 \int_{-\sqrt{y}}^{2-y} f(x,y)\, dxdy.$$

(a) Sketch the region R such that $I = \int\int_R f(x,y)\, dxdy$.
(b) Change the order of integration and re-write I as a single iterated integral

(c) Evaluate

$$J = \int_0^1 \int_{\sqrt{y}}^1 y^2 \ln(1 + x^7) dx dy.$$

[12] **7**. Let T denote the solid bounded by the coordinate planes $x = 0$, $y = 0$, $z = 0$ and the plane $2x + y + z = 2$.
(a) Draw a rough sketch of T
(b) Evaluate K where

$$K = \iiint_T x\, dV.$$

[14] **8**. Let B be the body bounded from below by the cone $z^2 = x^2 + y^2$ and from above by the plane $z = 1$.
Let

$$J = \int\int\int_B \frac{z}{x^2 + y^2 + z^2}\, dV$$

(a) Draw a rough sketch of B.
(b) Write J as an iterated integral in (i) cylindrical and (ii) spherical coordinates.
(c) Evaluate the integral J.