THE UNIVERSITY OF BRITISH COLUMBIA
Sessional Examinations – April 2011

MATHEMATICS 215

TIME: 3 hours

NO AIDS ARE PERMITTED. Note that the maximum number of points is 67. A score of N/67 will be treated as N/55, If N ≥ 55, then your mark will be 55/55. Also note that this exam has **three** pages. The value of each question is indicated.

(5) 1. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{2y - x}{2x}. \qquad (1)$$

(5) 2. Find the general solution of the system of differential equations

$$\left.\begin{aligned} \frac{dx}{dt} &= -2x \\ \frac{dy}{dt} &= x - 2y \end{aligned}\right\}. \qquad (2)$$

(5) 3. Sketch the trajectories of the solutions of system (2) in the *xy*-phase plane for $-\infty < t < \infty$, indicating by arrows the direction of increasing t.

(3) 4. Sketch the $y(t)$ component of solutions of system (2).

(2) 5. Suppose $x = X(t)$, $y = Y(t)$, is the solution of system (2) satisfying the initial conditions $x(0) = 10.12223$, $y(0) = 7.41396$. Find $\lim_{t\to\infty} X(t)$, $\lim_{t\to\infty} Y(t)$.

6. Consider the initial value problem (IVP)

$$\left.\begin{aligned} \frac{dx}{dt} &= -2x + A, \\ \frac{dy}{dt} &= x - 2y + Be^{-2t}, \\ x(0) &= \alpha, \quad y(0) = \beta, \end{aligned}\right\} \qquad (3)$$

where α, β, A and B are constants.

(2) (a) Find $\lim_{t\to\infty} x(t)$, $\lim_{t\to\infty} y(t)$ for the solution of the IVP (3).

(b) When the constants $\alpha = \beta = 0$, solve the IVP (3) for the following cases.

(4) (i) $A = 1$, $B = 0$;

(4) (ii) $A = 0$, $B = 1$.

7. Consider the initial value problem (IVP)

$$x'' + x' = \begin{cases} 1, & 0 < t \le 1, \\ e^{-t}, & t \ge 1, \end{cases} \qquad (4)$$
$$x(0) = x'(0) = 0.$$

(14) (a) Use two different methods to solve the IVP (4).

(2) (b) Which method is better? Why?

(2) (c) Find $\lim_{t\to\infty} x(t)$ for the solution of the IVP (4).

(2) (d) Would the answer in part (c) change if one used different initial conditions? Give a reason for your answer.

8. Consider the system of differential equations

$$\left.\begin{aligned} \frac{dx}{dt} &= x^2 + y, \\ \frac{dy}{dt} &= 2x + y. \end{aligned}\right\} \qquad (5)$$

(2) (a) Find the critical points of the system (5).

(10) (b) Near each of its critical points, sketch the trajectories of the solutions of the approximating linear system in the *xy*-phase plane, indicating by arrows the direction of increasing t.

(5) (c) Sketch trajectories of the solutions of system (5) in the *xy*-phase plane for $-\infty < t < \infty$, indicating by arrows the direction of increasing t.

TABLE OF INFORMATION

FUNCTION	**LAPLACE TRANSFORM**
$f(t)$	$L\{f\{t)\} = F(s)$
$u_a(t)$	$\frac{e^{-as}}{s}$
$u_a(t)f(t-a)$	$e^{-as}F(s)$
$\sin t$	$\frac{1}{s^2+1}$
$\cos t$	$\frac{s}{s^2+1}$
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
$tf(t)$	$-F'(s)$
$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$
$\delta(t-a)$	e^{-as}
$e^{at}f(t)$	$F(s-a)$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-[sf(0)+f'(0)]$