The University of British Columbia

Final Examination - December 10, 2012

Mathematics 200

All Sections

Closed book examination — Time: 2.5 hours

Last Name: ____________________ , **First:** ______________ **Signature** ____________________

Student Number ____________________

Special Instructions:

No books, notes or calculators are allowed.

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  (a) speaking or communicating with other candidates, unless otherwise authorized;
  (b) purposely exposing written papers to the view of other candidates or imaging devices;
  (c) purposely viewing the written papers of other candidates;
  (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

[10] **1**. Let L be the line of intersection of the planes $x + y + z = 6$ and $x - y + 2z = 0$.

(i) Find the points in which the line L intersects the coordinate planes.

(ii) Find parametric equations for the line through the point $(10, 11, 13)$ that is perpendicular to the line L and parallel to the plane $y = z$.

[10] **2**. Assume that the function $F(x, y, z)$ satisfies the equation $\dfrac{\partial F}{\partial z} = \dfrac{\partial^2 F}{\partial x^2} + \dfrac{\partial^2 F}{\partial y^2}$ and the mixed partial derivatives $\dfrac{\partial^2 F}{\partial x \partial y}$ and $\dfrac{\partial^2 F}{\partial y \partial x}$ are equal. Let A be some constant and let $G(\gamma, s, t) = F(\gamma + s, \gamma - s, At)$. Find the value of A such that $\dfrac{\partial G}{\partial t} = \dfrac{\partial^2 G}{\partial \gamma^2} + \dfrac{\partial^2 G}{\partial s^2}$.

[10] **3**. Suppose that a function $z = f(x, y)$ is implicitly defined by an equation: $xyz + x + y^2 + z^3 = 0$.

(i) Find $\dfrac{\partial z}{\partial x}$.

(ii) If $f(-1, 1) < 0$, find the linear approximation of the function $z = f(x, y)$ at $(-1, 1)$.

(iii) If $f(-1, 1) < 0$, use the linear approximation in (ii) to approximate $f(-1.02, 0.97)$.

[10] **4.** Find the absolute maximum and minimum values of the function $f(x, y) = 5 + 2x - x^2 - 4y^2$ on the rectangular region $R = \{(x, y) \mid -1 \leq x \leq 3, -1 \leq y \leq 1\}$.

[10] **5**. The directional derivative of a function $w = f(x, y, z)$ at a point P in the direction of the vector $\vec{i}$ is 2, in the direction of the vector $\vec{i} + \vec{j}$ is $-\sqrt{2}$, and in the direction of the vector $\vec{i} + \vec{j} + \vec{k}$ is $-\frac{5}{\sqrt{3}}$. Find the direction in which the function $w = f(x, y, z)$ has the maximum rate of change at the point P. What is this maximum rate of change?

[10] **6.**

(i) Use Lagrange multipliers to find the extreme values of

$$f(x, y, z) = (x-2)^2 + (y+2)^2 + (z-4)^2$$

on the sphere $x^2 + y^2 + z^2 = 6$.

(ii) Find the point on the sphere $x^2 + y^2 + z^2 = 6$ that is farthest from the point $(2, -2, 4)$.

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[10] **7**. Let

$$I = \int_0^4 \int_{\sqrt{y}}^{\sqrt{8-y}} f(x,y)\, dx\, dy.$$

(i) Sketch the domain of integration.

(ii) Reverse the order of integration.

(iii) Evaluate the integral for $f(x,y) = \dfrac{1}{(1+y)^2}$.

Hints: You may use $\dfrac{1}{9-x^2} = \dfrac{1}{6}\left(\dfrac{1}{x+3} - \dfrac{1}{x-3}\right)$.

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[10] **8**. A metal crescent is obtained by removing the interior of the circle defined by the equation $x^2 + y^2 = x$ from the metal plate of constant density 1 occupying the unit disc $x^2 + y^2 \leq 1$.

(i) Find the total mass of the crescent.

(ii) Find the x-coordinate of its center of mass.

Hint: you may use the fact that $\displaystyle\int_{-\pi/2}^{\pi/2} \cos^4(\theta)d\theta = \frac{3\pi}{8}$.

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[10] **9**. Evaluate $\iiint_R yz^2 e^{-xyz}\, dV$, over the rectangular box

$$R = \{(x, y, z) \mid 0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3\}.$$

[10] **10**. Evaluate

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{1-\sqrt{1-x^2-y^2}}^{1+\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^{\frac{5}{2}}\, dz\, dy\, dx$$

by changing to spherical coordinates.