The University of British Columbia

Final Examination - December, 2009

Mathematics 200

Closed book examination | Time: 2.5 hours

Last Name: ________________, **First:** ____________ **Signature** ________________

Student Number ________________

Special Instructions:

No books, notes or calculators are allowed.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		12
2		10
3		12
4		14
5		10
6		14
7		14
8		14
Total		100

[12] **1**. A surface is defined implicitly by $z^4 - xy^2z^2 + y = 0$.

(i) Compute $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ in terms of x, y, z.

(ii) Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(x, y, z) = (2, -1/2, 1)$.

(iii) If x decreases from 2 to 1.94, and y increases from -0.5 to -0.4,

find the approximate change in z from 1.

(iv) Find the equation of the tangent plane to the surface at the point $(2, -1/2, 1)$.

[10] **2**. For the surface

$$z = f(x, y) = x^3 + xy^2 - 3x^2 - 4y^2 + 4.$$

Find and classify [as local maxima, local minima, or saddle points] all critical points of $f(x, y)$

[12] **3**. The temperature $T(x, y)$ at a point of the xy-plane is given by

$$T(x, y) = 20 - 4x^2 - y^2.$$

(i) Find the maximum and minimum values of $T(x, y)$ on the disk D defined by $x^2 + y^2 \leq 4$.

(ii) Suppose an ant lives on the disk D. If the ant is initially at point $(1, 1)$, in which direction should it move so as to increase its temperature as quickly as possible?

(iii) Suppose that the ant moves at a velocity $\mathbf{v} = \langle -2, -1 \rangle$. What is its rate of increase of temperature as it passes through $(1, 1)$?

(iv) Suppose the ant is constrained to stay on the curve $y = 2 - x^2$. Where should the ant go if it wants to be as warm as possible?

[14] **4.** Use Lagrange multipliers to find the minimum distance from the origin to all points on the intersection of the curves

$$\begin{aligned} g(x,y,z) &= x - z - 4 = 0 \\ \text{and} \quad & h(x,y,z) = x + y + z - 3 = 0. \end{aligned}$$

[10] **5**. Find the volume (V) of the solid bounded above by the surface

$$z = f(x, y) = e^{-x^2},$$

below by the plane $z = 0$ and over the triangle in the x, y plane formed by the lines $x = 1$, $y = 0$ and $y = x$.

[14] **6**. For the integral $I = \int_0^1 \int_y^{2-y} \frac{y}{x}\, dxdy$.

(i) Sketch the region of integration.

(ii) Interchange the order of integration.

(iii) Evaluate I.

[14] **7**. A thin plate of uniform density 1 is bounded by the positive x and y axes and the cardioid $\sqrt{x^2 + y^2} = r = 1 + \sin\theta$, which is given in polar coordinates.
Find the x coordinate of its centre of mass.

[14] **8**. Let

$$I = \iiint_T xz \, dV.$$

where T is the eighth of the sphere $x^2 + y^2 + z^2 \leq 1$ with $x, y, z \geq 0$.

(i) Express I as a triple integral in spherical coordinates.

(ii) Express I as a triple integral in cylindrical coordinates.

(iii) Evaluate I by any method.