

Final Exam Duration: 2.5 hours

This test has 10 questions on 15 pages, for a total of 85 points.

- Read all the questions carefully before starting to work.
- All questions are long-answer; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: _____ Last Name: _____

Student-No: _____

Signature: _____

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	8	8	8	19	6	10	7	7	6	6	85
Score:											

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (i) speaking or communicating with other examination candidates, unless otherwise authorized;
 - (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
 - (iii) purposely viewing the written papers of other examination candidates;
 - (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Justify your answers and show all your work. Unless otherwise indicated simplification of answers is not required.

1. Consider the following function

$$f(x) = 2x^4 + \frac{1}{x} - \sqrt[3]{x^2} + 3e.$$

4 marks

- (a) Compute the derivative of the function $f(x)$.

4 marks

- (b) Compute the general anti-derivative of the function $f(x)$.

4 marks

2. (a) Find the equation of the tangent line to

$$f(x) = e^x \cos x$$

at the point $x = \frac{\pi}{2}$.

4 marks

- (b) Find a function $g(x)$ satisfying $g(\pi/4) = 1$ such that

$$g'(x) = \sqrt{2} \sin x + 3 \cos(2x).$$

4 marks

3. (a) Find the derivative of

$$f(x) = \ln(\sin(x^2)).$$

4 marks

(b) Compute

$$\int_{-1}^3 g(x) dx$$

where

$$g(x) = \begin{cases} 1, & x > 0 \\ e^x, & x \leq 0 \end{cases}.$$

4. Compute the following integrals.

4 marks

(a)

$$\int_1^4 \frac{\sqrt{x} + x^2}{x} dx$$

5 marks

(b)

$$\int x^2 \sin(x^3 + 1) dx$$

5 marks

(c)

$$\int (2x + 1) \sin x \, dx$$

5 marks

(d)

$$\int \ln x \, dx$$

6 marks

5. You are currently 3 meters East of a flag pole and move towards the pole at 1 meter per second. Your friend is 4 meters North of the pole and moves away from it at 2 meters per second. How fast is the distance between you and your friend changing? Is the distance increasing or decreasing?

4 marks

6. (a) Explain using a picture why the expression

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

should give exactly the area under the curve $f(x)$. Indicate on your picture what n , x_i , $f(x_i)$ and Δx are.

6 marks

(b) Approximate the following integral using Riemann Sums

$$\int_1^{2.5} 2x \, dx.$$

Use left endpoints and $n = 3$ (ie. four bars). Is your approximation less than, greater than, or exactly equal to the true value of the integral? Explain why.

7. After Matt graduates he's going to spend the summer scooping ice cream cones. On his first day the *rate of change* of the number of ice cream cones he produces is

$$r(t) = 5 - \frac{4}{(t+1)^2}.$$

in units of cones/minute.

6 marks

- (a) Assuming that at $t = 0$ he has produced no cones, how many cones will he have produced after his first 3 minutes?

1 mark

- (b) After a long period of time Matt will get good at scooping ice cream. Eventually, about how many cones will he be able to produce per minute?

3 marks

8. (a) Suppose that

$$\int_1^{-1} f(x) dx = 4 \quad \text{and} \quad \int_1^2 f(x) dx = 3.$$

Find

$$\int_{-1}^2 f(x) dx.$$

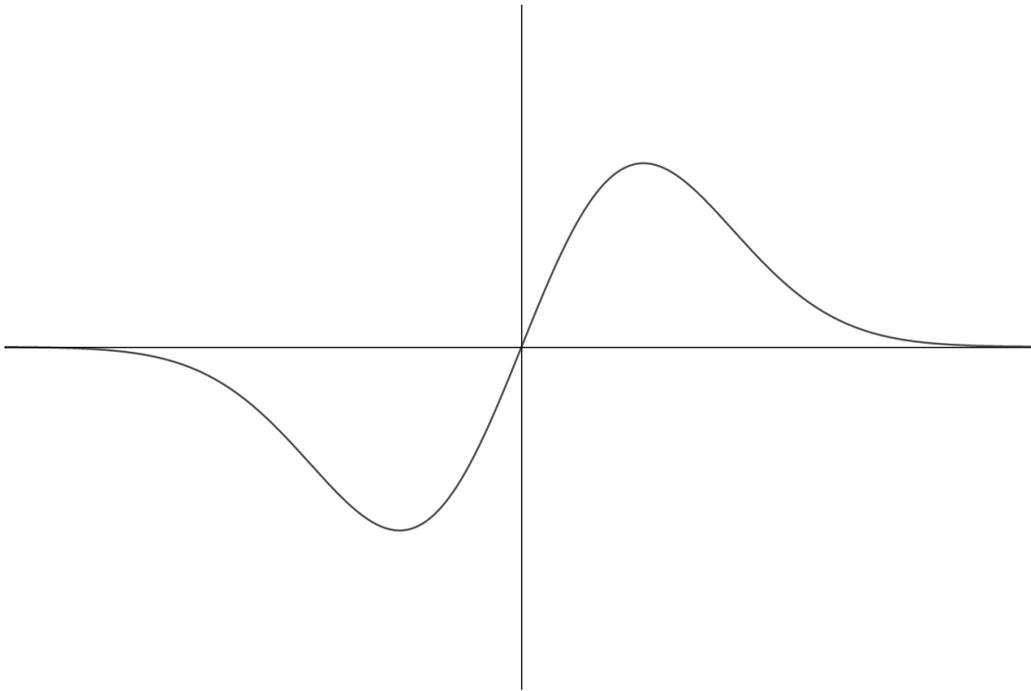
4 marks

(b) Find *all* values of b such that

$$\int_3^b e^{7x} dx = 0.$$

Ensure you justify your answer fully.

9. Observe the following graph of $f(x)$



1 mark

(a) Is $f(x)$ an even or odd function?

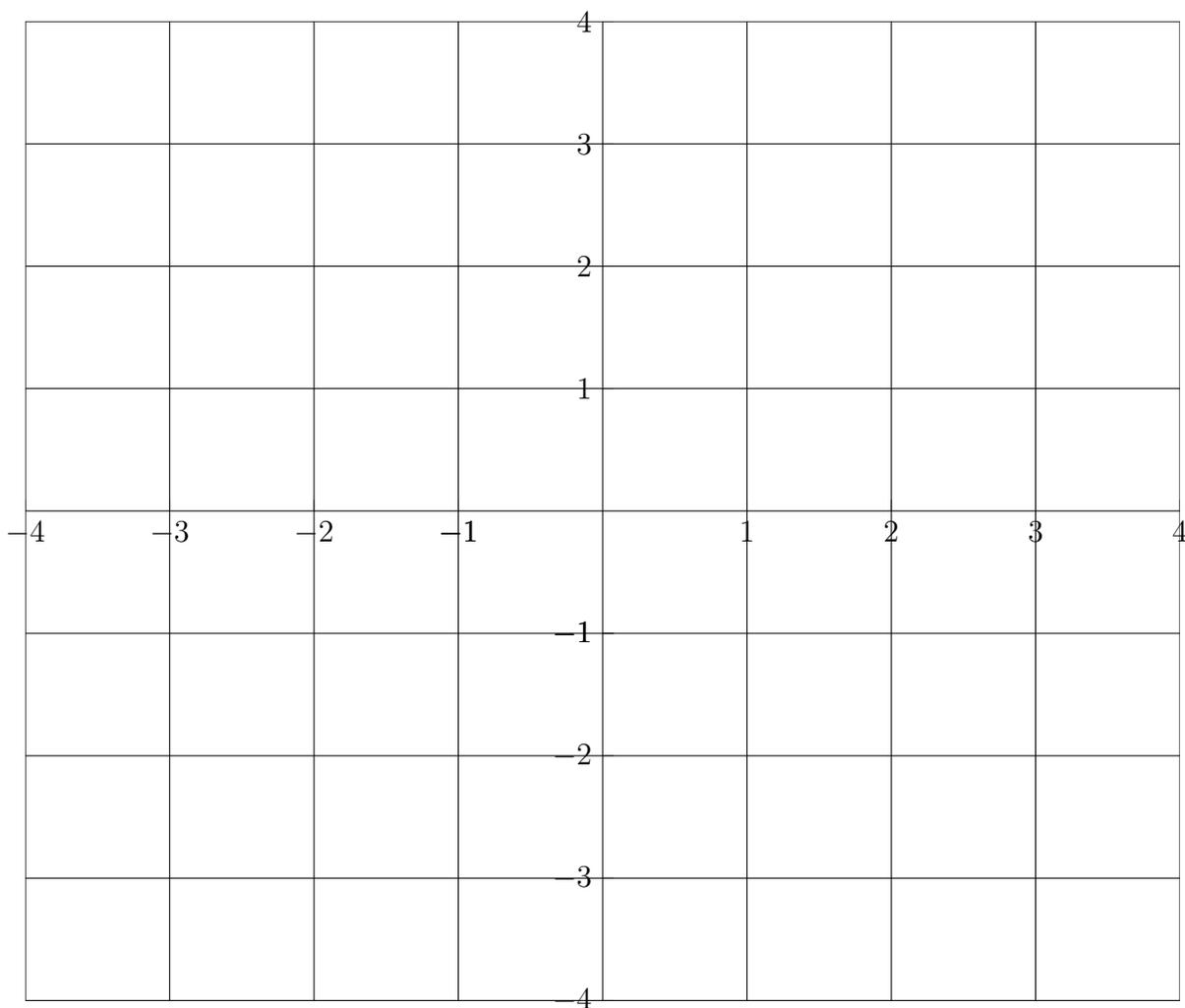
5 marks

(b) Sketch the graph of the derivative of $f(x)$. Is $f'(x)$ even or odd?

6 marks 10. Sketch the graph of a function with domain $f(x)$ is $\{x \in \mathbb{R} : -4 \leq x \leq 4\}$ satisfying the following properties:

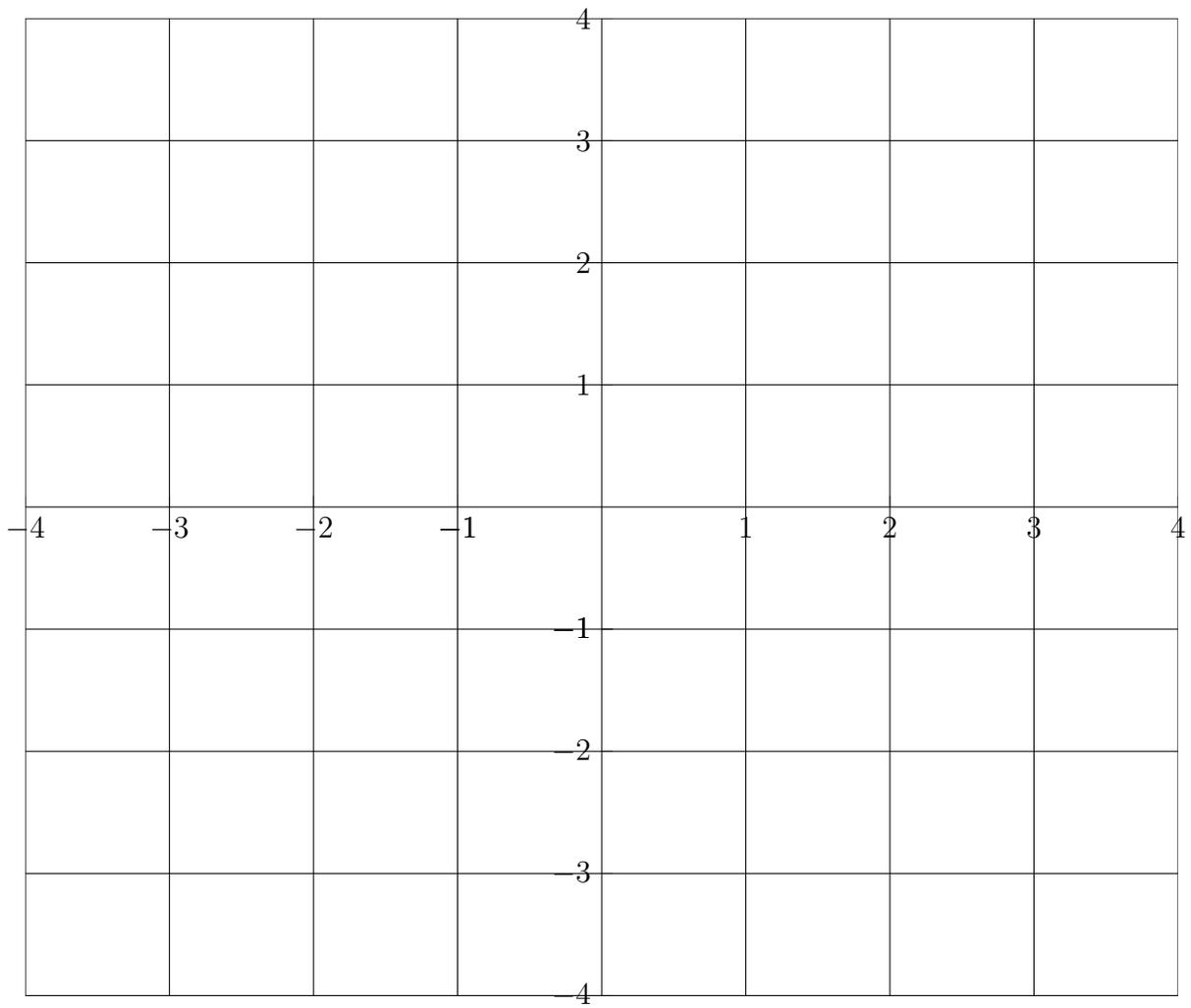
- $f(-3) > 0$
- $f(1) < 0$
- $f'(-2) = 0$
- $\int_{-4}^0 f(x) dx = 0$
- $\int_0^4 f(x) dx > 0$
- $\lim_{x \rightarrow 3} f'(x) = \infty$

You do not need to find an equation for your function. Use the axes below.



There is another set of axes on the following page (in case you ruin the first one).

Extra axes:



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