**The University of British Columbia**

Final Examination - April 26, 2008

**Mathematics 152**

All Sections

Closed book examination. No calculators. Time: 2.5 hours

**Last Name** ____________ **First** ____________ **Signature** ____________

**Section** : ____________

**Student Number** ____________

**Instructor** : ____________

## Special Instructions:

No books, notes, or calculators are allowed. Show all your work, little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page.

### Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practises shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

| | | |
|---|---|---|
| 1 | | 5 |
| 2 | | 5 |
| 3 | | 5 |
| 4 | | 5 |
| 5 | | 5 |
| 6 | | 5 |
| 7 | | 5 |
| 8 | | 5 |
| 9 | | 5 |
| 10 | | 5 |
| 11 | | 5 |
| 12 | | 5 |
| Total | | 60 |

[5] **1**. Find the equation of the plane passing through the points (1,1,1), (1,-2,0), and (0,0,3).

[5] **2**. $T$ is a linear transformation from $R^2$ to $R^2$ with matrix representation

$$\mathbf{T} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

**(a)** [1] If $\mathbf{x} = [1, 2]^T$ what is $T(\mathbf{x})$?

**(b)** [3] For which vector $\mathbf{y}$ is $T(\mathbf{y}) = [1, 2]^T$?

**(c)** [1] Describe the action of $T$ geometrically.

[5] **3**. Find all solutions of

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_1 - 2x_2 + x_3 &= 0. \end{aligned}$$

[5] **4**. Find the determinant of

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 0 & 6 & 7 \\ 0 & 4 & 1 & 2 \\ 3 & 0 & 8 & 9 \end{bmatrix}$$

[5] **5**. Find the inverse of

$$\begin{bmatrix} 2 & 1 & 3 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

[5] **6**. Find the solution $\mathbf{x}(t) = [x_1(t), x_2(t)]^T$ to the differential equation system

$$\begin{aligned} \frac{dx_1}{dt} &= x_1 + 2x_2 \\ \frac{dx_2}{dt} &= 2x_1 + x_2 \end{aligned}$$

with $x_1(0) = 1$ and $x_2(0) = 2$.

[5] **7**. Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} 0 & -4 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

[5] **8**. Consider the system of equations for $\mathbf{x} = [x_1, x_2]^T$:

$$\begin{aligned} x_1 + 2x_2 &= 1 \\ x_1 - x_2 &= 5 \\ 2x_1 + 3x_2 &= 1 \end{aligned}$$

**(a)** [1] Show that the system has no solutions.

**(b)** [3] Find the least squares solution.

**(c)** [1] Write the quadratic function in $x_1$ and $x_2$ that your solution in part (b) above minimizes. You *do not need to simplify* your answer.

[5] **9**. Consider the transition matrix from a random walk given below:

$$\mathbf{P} = \begin{bmatrix} 1/2 & 1/3 \\ 1/2 & 2/3 \end{bmatrix}$$

Let $x_1^{(n)}$ and $x_2^{(n)}$ be the probabilities of being in states 1 and 2 respectively after $n$ steps of the walk.

**(a)** [2] If you begin in state 1 (that is, $x_1^{(0)} = 1$ and $x_2^{(0)} = 0$) what is $x_1^{(3)}$ (that is, the chance you will be in state 1 after 3 steps)?

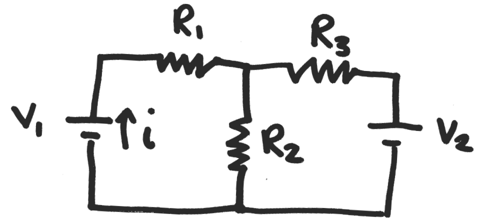
**(b)** [2] Find the eigenvalues and eigenvectors of $\mathbf{P}$.

**(c)** [1] If you begin in state 1 what is the chance that you will be in state 1 after many steps? That is, what is

$$\lim_{n\to\infty} x_1^{(n)}?$$

Your work in part (b) will be very helpful to answer this question.

[5] **10**. Consider the circuit below where $V_1 = 12$ V, $V_2 = 8$ V, $R_1 = 6\Omega$, $R_2 = 3\Omega$ and $R_3 = 2\Omega$. Find the current $i$ through the first voltage source as shown.



[5] **11**. The following MATLAB commands were typed in, followed by <enter>:

- `A = [[1 2 2];[2 1 2];[2 2 1]];`
- `x = [1 1 1];`
- `y = [1 2 3];`

What will be the result of the following MATLAB commands, followed by <enter>:

**(a)** [1] `x.*y`

**(b)** [1] `x*y`

**(c)** [1] `x*y'`

**(d)** [1] `A*x'`

**(e)** [1] `A\x'`

Recall that in MATLAB, the symbol <'> denotes transpose. *Hint:* one of the commands above will lead to an error message. For this case, describe why an error occurs.

[5] **12**. Give answers to each of the following questions with a *brief* discussion of your reasoning.

**(a)** If 2 and 3+i are eigenvalues of a $3 \times 3$ matrix, what are the possible values for other eigenvalues?

**(b)** If an $n \times n$ matrix $\mathbf{A}$ is invertible, what is its rank?

**(c)** Give an example of a 4 by 4 matrix with determinant zero *and* no zero entries.

**(d)** If $\mathbf{A}$ and $\mathbf{T}$ are $n \times n$ matrices with $\mathbf{T}$ invertible and the determinant of $\mathbf{A}$ equal to 1, what is the determinant of $\mathbf{T}^{-1}\mathbf{A}\mathbf{T}$?

**(e)** Are the vectors (1,2,3), (1,2,4), (2,7,1), and (0,0,1) linearly independent?

**The End**