

The University of British Columbia

Final Examination - April 18, 2016

Mathematics 105

All Sections

Closed book examination

Time: 2.5 hours

Last Name _____ First _____ Signature _____

Student Number _____ Section Number _____ Instructor _____

Special Instructions:

No books, notes, or calculators are allowed. A formula sheet is included.

Senate Policy: Conduct during examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		42
2		15
3		10
4		14
5		13
6		6
Total		100

[42] 1. **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

- (a) Find an equation of the plane that passes through the point $(-1, 0, 3)$ with a normal vector $\langle 3, 5, -2 \rangle$.

Answer:

- (b) Find the domain of the function $f(x, y) = \ln(4 - x^2 - y^2)$.

Answer:

- (c) Let $f(x, y) = x \sin(xy)$. Find $f_{xx} = \frac{\partial^2 f}{\partial x^2}$.

Answer:

- (d) Determine whether the following statement is true (you do not need to justify your answer) :

The function $f(x, y)$ could have both an absolute maximum and an absolute minimum at two different points that are not critical points.

Answer:

- (e) Does a left Riemann sum underestimate or overestimate the area of the region under the graph of a function that is positive and decreasing on an interval $[a, b]$? You do not need to justify your answer.

Answer:

- (f) Evaluate $\int_{-1}^5 f(x) dx$, where $f(x) = \begin{cases} 2x + 4 & \text{if } x \geq 3 \\ 10 & \text{if } x < 3 \end{cases}$.

Answer:

- (g) Solve the differential equation: $y'(t) = -0.03y + 2$. You do not need to express your solution as a function of t explicitly.

Answer:

- (h) Use substitution rule to find the indefinite integral $\int x^5(x^6 + 23)^{99} dx$.

Answer:

- (i) Evaluate $\int \frac{\ln x}{x^9} dx$.

Answer:

(j) Evaluate $\int \cos^3 x \sin^{-2} x dx$.

Answer:

(k) Evaluate $\int \sqrt{16 - x^2} dx$. Your answer must be in terms of x , but it is acceptable to leave in inverse trigonometric functions.

Answer:

- (l) Let R be the region bounded by the graph of $y = e^{-3x}$ and the x -axis on the interval $[a, \infty)$, where a is a constant.. Find the value of a such that the area of the region R is 1.

Answer:

(m) Evaluate $\sum_{k=0}^{\infty} \left(2 \left(\frac{3}{4} \right)^k + \frac{5}{7^k} \right)$.

Answer:

- (n) Find the first three nonzero terms of the Taylor series for the function $f(x) = \frac{1}{(1+x^5)}$ centered at 0.

Answer:

Full-Solution Problems. In questions 2 – 6, justify your answers and show all your work.

[15] **2.**

[3] (a) Compute $\frac{d}{dx} \left(\int_x^{x^2} \sin(t^2) dt \right)$.

[3] (b) Give the right Riemann sum for $f(x) = x^{\frac{3}{2}}$ on the interval $[2, 6]$ with n subintervals of equal length.

2. [5] (c) Evaluate $\int \frac{81}{x^3 - 9x^2} dx$.

[4] (d) Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x \sqrt{\tan x} dx$.

[10] **3.** Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = y^2 - 4x^2$ subject to the constraint $x^2 + 2y^2 = 4$.

[14] 4.

(a) Suppose that a continuous random variable X has the probability density function

$$f(x) = \begin{cases} ax^2 & \text{if } 2 \leq x \leq 3 \\ \frac{12}{31} & \text{if } 1 \leq x < 2 \end{cases}$$

where a is a constant. Find the value of a .

(b) Let $f(x) = \sum_{k=1}^{\infty} \frac{x^k}{k 3^k}$, $g(x) = f'(x) = \frac{df}{dx}$, and R be the radius of convergence of the power series $f(x)$. Find R and $g(1)$.

[13] 5. Determine whether the following series converge or diverge.

[2] (a) $\sum_{k=2}^{\infty} \frac{k^2 + 2k + 1}{3k^2 + 1}$

[3] (b) $\frac{1}{2^2} + \frac{2}{3^2} + \frac{3}{4^2} + \cdots$

5. [3](c) $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$.

[5](d) $\sum_{k=1}^{\infty} \frac{\pi^{k+1}}{k^{\pi} k!}$.

[6] **6.** Evaluate $\int \frac{dx}{x^{2016} - x}$.

MATH 105 Exam Formula Sheet• **Summation formulas:**

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

• **Trigonometric formulas:**

$$\cos^2 x = \frac{1 + \cos(2x)}{2}, \quad \sin^2 x = \frac{1 - \cos(2x)}{2}, \quad \sin(2x) = 2 \sin x \cos x$$

• **Derivatives of some inverse trigonometric functions:**

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

• **Indefinite integrals:**

$$\int \sec x dx = \ln |\sec x + \tan x| + C, \quad \int \frac{dx}{1+x^2} = \tan^{-1} x + C = \arctan x + C$$

• **Probability:**

If X is a continuous random variable with probability density function $f(x)$ with $-\infty < x < \infty$, then the expected value $\mathbf{E}(X)$ is given by

$$\mathbf{E}(X) = \int_{-\infty}^{\infty} x f(x) dx$$

• **Some commonly used Taylor series centered at 0:**

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad \text{for } |x| < 1$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \text{for } |x| < \infty$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad \text{for } |x| < \infty$$

$$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \quad \text{for } 1 \geq |x|$$

• **Two important limits:**

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \quad \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

The End