The University of British Columbia

Final Examination - April 24, 2014

Mathematics 105

All Sections

Closed book examination Time: 2.5 hours

Last Name __________ **First** __________ **Signature** __________

Student Number __________ **Section Number** __________ **Instructor** __________

Special Instructions:

No books, notes, or calculators are allowed. A formula sheet is included.

1a,b,c,d,e		15
1f,g,h,i,j		15
1k,l,m,n		12
2		12
3		12
4		14
5		12
6		8
Total		100

[42] **1. Short-Answer Questions**. Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

(a) Let $Q : -x + 5y - 3z = 2$ and $R : 3x - \frac{9}{5}y - 4z = 0$ be two planes. Determine if Q and R are parallel, orthogonal, or identical.

Answer:

(b) The volume of a right circular cone of radius x and height y is $V(x, y) = \frac{\pi x^2 y}{3}$. Graph the level curve $V(x, y) = \pi$.

Answer:

(c) Let $f(x, y) = \sin(xy)$. Find $\frac{\partial^2 f}{\partial x \partial y}$.

Answer:

(d) Use sigma notation to write the midpoint Riemann sum for $f(x) = x^8$ on $[5, 15]$ with $n = 50$. Do not evaluate the midpoint Riemann sum.

Answer:

(e) Evaluate $\displaystyle\int_1^5 f(x)\,dx$, where $f(x) = \begin{cases} 3 & \text{if } x \leq 3 \\ x & \text{if } x \geq 3 \end{cases}$.

Answer:

(f) If $f'(1) = 2$ and $f'(2) = 3$, find $\displaystyle\int_1^2 f'(x)f''(x)\,dx$.

Answer:

(g) Evaluate $\int \cos^{-1} y \, dy$.

Answer:

(h) Evaluate $\int \cos^3 x \sin^4 x \, dx$.

Answer:

(i) Evaluate $\int \frac{dx}{\sqrt{3 - 2x - x^2}}$.

Answer:

(j) Evaluate $\displaystyle\int \frac{x-13}{x^2-x-6}\,dx$.

Answer:

(k) The random variable X has probability density function

$$f(x) = \begin{cases} 0 & \text{if } x < 1, \\ \frac{3}{2}x^{-5/2} & \text{if } x \geq 1. \end{cases}$$

Find the expected value $\mathbb{E}(X)$ of the random variable X.

Answer:

(l) Evaluate $\displaystyle\sum_{n=1}^{\infty}\left[\left(\frac{1}{3}\right)^{n}+\left(-\frac{2}{5}\right)^{n-1}\right]$.

Answer:

(m) Let k be a constant. Find the value of k such that $f(x) = 1 + k|x|$ is a probability density function on $-1 \leq x \leq 1$.

Answer:

(n) Let $h(s)$ be a continuous function with $h(10) = 2$. If $f(x, y) = \displaystyle\int_{1}^{xy} h(s)\,ds$, find $f_x(2, 5)$.

Answer:

Full-Solution Problems. In questions 2 – 6, justify your answers and show all your work.

[12] **2**. (a) Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt[3]{k^4+1}}{\sqrt{k^5+9}}$ converges.

(b) Find the radius of convergence of the power series $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{10^{k+1}(k+1)!}$.

2. (c) Let $\sum_{n=0}^{\infty} b_n x^n$ be the Maclaurin series for $f(x) = \dfrac{3}{x+1} - \dfrac{1}{2x-1}$, i.e., $\sum_{n=0}^{\infty} b_n x^n = \dfrac{3}{x+1} - \dfrac{1}{2x-1}$. Find b_n.

[12] **3**.

(a) Use the method of Lagrange multipliers to find the maximum and minimum values of $(x+1)^2+(y-2)^2$ on the circle $x^2+y^2=125$. A solution that does not use the method of Lagrange multipliers will receive no credit, even if the answer is correct.

3.(b) Find the point on the circle $x^2 + y^2 = 125$ that has minimum distance from the point $(-1, 2)$.

[14] **4**. Let $T(x, y) = \frac{1}{9}y^3 + x^2 - 2xy + 6x - 6y$.

(a) Find all critical points of $T(x, y)$ and classify each as a local maximum, local minimum or saddle point.

4.(b) Find the maximum and minimum values of $T(x, y)$ on the region $R = \left\{(x, y) : \frac{1}{3}y^2 - 3 \leq x \leq 0\right\}$.

[12] **5**. An endowment is an investment account in which the balance ideally remains constant and withdrawals are made on the interest earned by the account. Such an account may be modeled by the initial value problem $B'(t) = aB - m$ for $t \geq 0$, with $B(0) = B_0$. The constant a reflects the annual interest rate, m is the annual rate of withdrawal, and B_0 is the initial balance in the account.

(a) Solve the initial value problem with $a = 0.02$ and $B(0) = B_0 = \$30,000$. Note that your answer depends on the constant m.

5.(b) If $a = 0.02$ and $B(0) = B_0 = \$30,000$, what is the annual withdrawal rate m that ensures a constant balance in the account?

[8] **6**.

(a) Evaluate the sum of the convergent series

$$\sum_{k=1}^{\infty} \frac{1}{\pi^k k!}.$$

6.(b) Assume that the series $\sum_{n=1}^{\infty} \frac{na_n - 2n + 1}{n+1}$ converges, where $a_n > 0$ for $n = 1, 2, \cdots$. Is the following series

$$-\ln a_1 + \sum_{n=1}^{\infty} \ln\left(\frac{a_n}{a_{n+1}}\right)$$

convergent? If your answer is NO, justify your answer. If your answer is YES, evaluate the sum of the series $-\ln a_1 + \sum_{n=1}^{\infty} \ln\left(\frac{a_n}{a_{n+1}}\right)$.

The End