

The University of British Columbia

Final Examination - April 24, 2014

Mathematics 105

All Sections

Closed book examination

Time: 2.5 hours

Last Name _____ First _____ Signature _____

Student Number _____ Section Number _____ Instructor _____

Special Instructions:

No books, notes, or calculators are allowed. A formula sheet is included.

Senate Policy: Conduct during examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

| | | |
|------------|--|-----|
| 1a,b,c,d,e | | 15 |
| 1f,g,h,i,j | | 15 |
| 1k,l,m,n | | 12 |
| 2 | | 12 |
| 3 | | 12 |
| 4 | | 14 |
| 5 | | 12 |
| 6 | | 8 |
| Total | | 100 |

[42] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

- (a) Let $Q : -x + 5y - 3z = 2$ and $R : 3x - \frac{9}{5}y - 4z = 0$ be two planes. Determine if Q and R are parallel, orthogonal, or identical.

Answer:

- (b) The volume of a right circular cone of radius x and height y is $V(x, y) = \frac{\pi x^2 y}{3}$. Graph the level curve $V(x, y) = \pi$.

Answer:

- (c) Let $f(x, y) = \sin(xy)$. Find $\frac{\partial^2 f}{\partial x \partial y}$.

Answer:

- (d) Use sigma notation to write the midpoint Riemann sum for $f(x) = x^8$ on $[5, 15]$ with $n = 50$. Do not evaluate the midpoint Riemann sum.

Answer:

- (e) Evaluate $\int_1^5 f(x) dx$, where $f(x) = \begin{cases} 3 & \text{if } x \leq 3 \\ x & \text{if } x \geq 3 \end{cases}$.

Answer:

- (f) If $f'(1) = 2$ and $f'(2) = 3$, find $\int_1^2 f'(x)f''(x) dx$.

Answer:

(g) Evaluate $\int \cos^{-1} y \, dy$.

Answer:

(h) Evaluate $\int \cos^3 x \sin^4 x \, dx$.

Answer:

(i) Evaluate $\int \frac{dx}{\sqrt{3 - 2x - x^2}}$.

Answer:

(j) Evaluate $\int \frac{x - 13}{x^2 - x - 6} dx$.

Answer:

(k) The random variable X has probability density function

$$f(x) = \begin{cases} 0 & \text{if } x < 1, \\ \frac{3}{2}x^{-5/2} & \text{if } x \geq 1. \end{cases}$$

Find the expected value $\mathbb{E}(X)$ of the random variable X .

Answer:

(l) Evaluate $\sum_{n=1}^{\infty} \left[\left(\frac{1}{3}\right)^n + \left(-\frac{2}{5}\right)^{n-1} \right]$.

Answer:

(m) Let k be a constant. Find the value of k such that $f(x) = 1 + k|x|$ is a probability density function on $-1 \leq x \leq 1$.

Answer:

(n) Let $h(s)$ be a continuous function with $h(10) = 2$. If $f(x, y) = \int_1^{xy} h(s) ds$, find $f_x(2, 5)$.

Answer:

Full-Solution Problems. In questions 2 – 6, justify your answers and show all your work.

[12] **2.** (a) Determine whether the series $\sum_{k=1}^{\infty} \frac{\sqrt[3]{k^4 + 1}}{\sqrt{k^5 + 9}}$ converges.

(b) Find the radius of convergence of the power series $\sum_{k=0}^{\infty} \frac{x^k}{10^{k+1}(k+1)!}$.

2. (c) Let $\sum_{n=0}^{\infty} b_n x^n$ be the Maclaurin series for $f(x) = \frac{3}{x+1} - \frac{1}{2x-1}$, i.e., $\sum_{n=0}^{\infty} b_n x^n = \frac{3}{x+1} - \frac{1}{2x-1}$. Find b_n .

[12] **3.**

- (a) Use the method of Lagrange multipliers to find the maximum and minimum values of $(x+1)^2 + (y-2)^2$ on the circle $x^2 + y^2 = 125$. A solution that does not use the method of Lagrange multipliers will receive no credit, even if the answer is correct.

3.(b) Find the point on the circle $x^2 + y^2 = 125$ that has minimum distance from the point $(-1, 2)$.

[14] 4. Let $T(x, y) = \frac{1}{9}y^3 + x^2 - 2xy + 6x - 6y$.

- (a) Find all critical points of $T(x, y)$ and classify each as a local maximum, local minimum or saddle point.

4.(b) Find the maximum and minimum values of $T(x, y)$ on the region $R = \left\{ (x, y) : \frac{1}{3}y^2 - 3 \leq x \leq 0 \right\}$.

[12] **5.** An endowment is an investment account in which the balance ideally remains constant and withdrawals are made on the interest earned by the account. Such an account may be modeled by the initial value problem $B'(t) = aB - m$ for $t \geq 0$, with $B(0) = B_0$. The constant a reflects the annual interest rate, m is the annual rate of withdrawal, and B_0 is the initial balance in the account.

- (a) Solve the initial value problem with $a = 0.02$ and $B(0) = B_0 = \$30,000$. Note that your answer depends on the constant m .

5.(b) If $a = 0.02$ and $B(0) = B_0 = \$30,000$, what is the annual withdrawal rate m that ensures a constant balance in the account?

[8] **6.**

(a) Evaluate the sum of the convergent series

$$\sum_{k=1}^{\infty} \frac{1}{\pi^k k!}.$$

6.(b) Assume that the series $\sum_{n=1}^{\infty} \frac{na_n - 2n + 1}{n + 1}$ converges, where $a_n > 0$ for $n = 1, 2, \dots$. Is the following series

$$-\ln a_1 + \sum_{n=1}^{\infty} \ln \left(\frac{a_n}{a_{n+1}} \right)$$

convergent? If your answer is NO, justify your answer. If your answer is YES, evaluate the sum of the series $-\ln a_1 + \sum_{n=1}^{\infty} \ln \left(\frac{a_n}{a_{n+1}} \right)$.