

Math 105 Final Exam Formula Sheet

• **Trigonometric Formulas:**

$$\sin^2(x) + \cos^2(x) = 1 \qquad \tan^2(x) + 1 = \sec^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x) \qquad \cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x) \qquad \cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x)$$

$$\sin(x + y) = \sin(x) \cos(y) + \sin(y) \cos(x) \qquad \cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

• **Values of Trigonometric Functions:**

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	0

• **Derivatives of Inverse Trigonometric Functions:**

Notation: $\sin^{-1}(x) = \arcsin(x) \qquad \tan^{-1}(x) = \arctan(x) \qquad \sec^{-1}(x) = \operatorname{arcsec}(x)$

Derivatives: $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1} \qquad \frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

• **Numerical Integration:**

Midpoint Rule for $\int_a^b f(x)dx$, with $\Delta x = \frac{b-a}{n}$, $x_k = a + k\Delta x$:

$$M(n) = \sum_{k=1}^n f\left(\frac{x_{k-1} + x_k}{2}\right) \Delta x.$$

Trapezoid Rule for $\int_a^b f(x)dx$, with $\Delta x = \frac{b-a}{n}$, $x_k = a + k\Delta x$:

$$T(n) = \left(\frac{1}{2}f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2}f(x_n) \right) \Delta x.$$

• **Economics:**

$$\text{Consumer surplus} = \int_0^{q_e} D(q)dq - p_e q_e \qquad \text{Producer surplus} = p_e q_e - \int_0^{q_e} S(q)dq$$

$$\text{Future value of } S(t) = \int_0^T S(t)e^{r(T-t)}dt \qquad \text{Present value of } I(t) = \int_0^T I(t)e^{-rt}dt$$

• **Probability:**

Expected value: $E(X) = a_1p_1 + a_2p_2 + \dots + a_np_n$

Variance: $Var(X) = (a_1 - E(X))^2p_1 + (a_2 - E(X))^2p_2 + \dots + (a_n - E(X))^2p_n$