

Final Exam — December 18th 2015 Duration: 2.5 hours

This test has 11 questions on 14 pages, for a total of 100 points.

- Read all the questions carefully before starting to work.
- Q1-Q6 are short-answer questions[3pts each]; put your answer in the boxes provided.
- All other questions are long-answer; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)
- Please circle your course and section:

104	101	102	103	104	105	106	108	109
184	101	102	103	104	105	106	I don't know.	

First Name: _____ Last Name: _____

Student-No: _____ Signature: _____

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	9	6	9	9	9	6	10	10	10	12	10	100
Score:												

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (i) speaking or communicating with other examination candidates, unless otherwise authorized;
 - (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
 - (iii) purposely viewing the written papers of other examination candidates;
 - (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Short-Answer Questions. Put your answer in the box provided. Full marks will be given for a correct answer placed in the box, while part marks may be given for work shown. Unless otherwise stated, calculator ready answers are acceptable.

3 marks

1. (a) Compute $\lim_{x \rightarrow -2} \frac{2x^2 - 5x - 18}{x + 2}$.

Answer:

3 marks

(b) Suppose $f(2) = 3$ and $f'(2) = 2$. Let $h(x) = \frac{f(x)}{x^2}$. Find the equation of the line tangent to $y = h(x)$ at $x = 2$.

Answer:

3 marks

(c) Compute the derivative of $f(x) = \sin^2(e^x)$.

Answer:

3 marks

2. (a) If \$4,000 dollars is invested in a bank account at an interest rate of 6% per year, compounded continuously. How long will it take, in years, for the balance to reach \$20,000 dollars?

Answer:

3 marks

- (b) Find the absolute maximum of $f(x) = \ln(x^2 + x + 1)$ over the interval $[-1, 1]$.

Answer:

3 marks

3. (a) A (spherical) balloon is being deflated at the rate of $8 \text{ cm}^3/\text{s}$. How fast is its radius changing when the radius is 2 cm ? *Note: The volume V of a sphere of radius r is $\frac{4\pi}{3}r^3$.*

Answer:

3 marks

- (b) Find the value of a for which the function $f(x)$ is continuous for all x .

$$f(x) = \begin{cases} x^2 + a & \text{when } x \leq e, \\ 3a \ln(x) & \text{when } x > e. \end{cases}$$

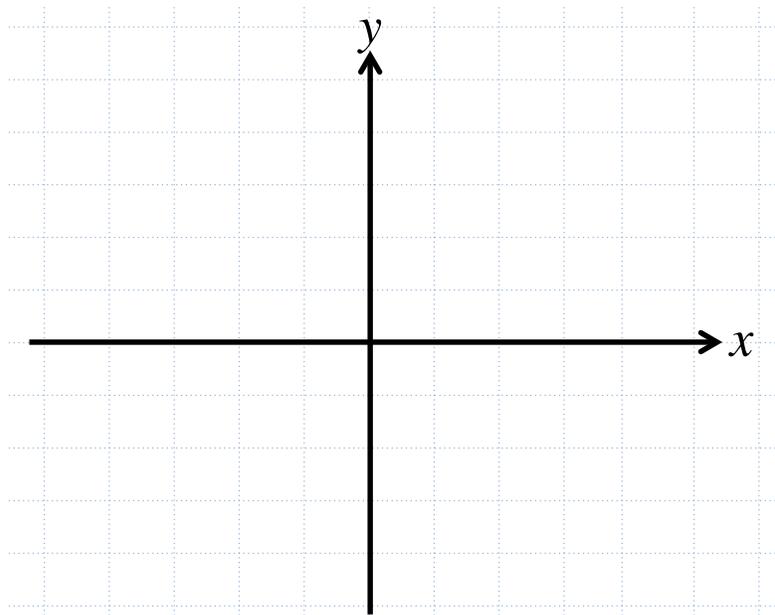
Answer:

3 marks

- (c) Sketch a graph of a continuous function $y = f(x)$ that satisfies the following:

- $f(-5) = f(0) = 0$;
- $f'(x) < 0$ for $-5 < x < -3$ and $1 < x < 5$;
- $f'(x) > 0$ for $-3 < x < 1$;
- $f''(x) < 0$ for $-2 < x < 3$;
- $f''(x) > 0$ for $-5 < x < -2$ and $3 < x < 5$;

Clearly label and local extrema and points of inflection.



3 marks

4. (a) Compute the 3rd degree Taylor polynomial of $f(x) = e^{-3x}$ at $a = 0$.

Answer:

3 marks

- (b) Determine an equation of the line tangent to the curve

$$x^4 - x^2y + y^4 = 1$$

at the point $(-1, 1)$.

Answer:

3 marks

- (c) When a company manufactures x desks, the per-desk *average cost* is given by the function

$$\bar{C}(x) = \frac{50}{x} + \frac{7}{\sqrt{x}} - \frac{100}{x2^x}.$$

Use this information to determine the marginal cost function.

Answer:

3 marks

5. (a) Use linear approximation to estimate the value of $\sqrt[3]{30}$. State your answer accurate to two decimal places.

Answer:

3 marks

- (b) We want to estimate a solution to the equation

$$2^x = x + \pi$$

Find a positive integer n such that a solution lies in the interval $[n, n + 1]$.

Answer:

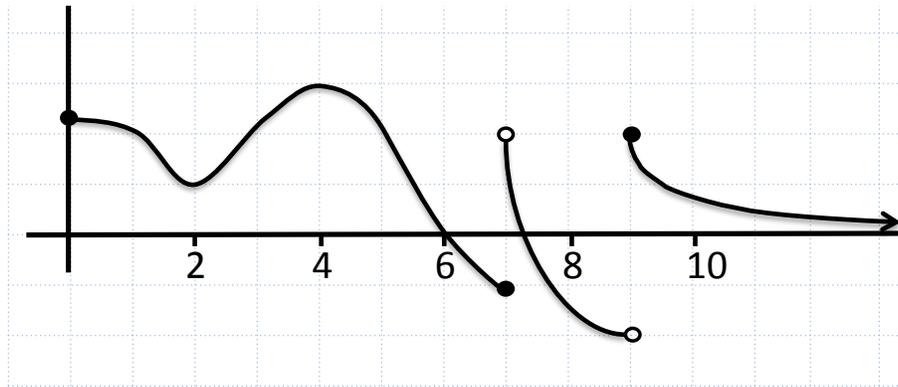
3 marks

- (c) A tailor is currently producing 80 suits per month and sells them for \$100 per suit. His monthly demand curve is given by $q = 100 - 2\sqrt{p}$. Find the current price elasticity of demand and use it to decide whether price should be raised or lowered to increase his revenue.

Answer:

3 marks

6. (a) Below is a graph of $f(x)$.



i. What is the global maximum value?

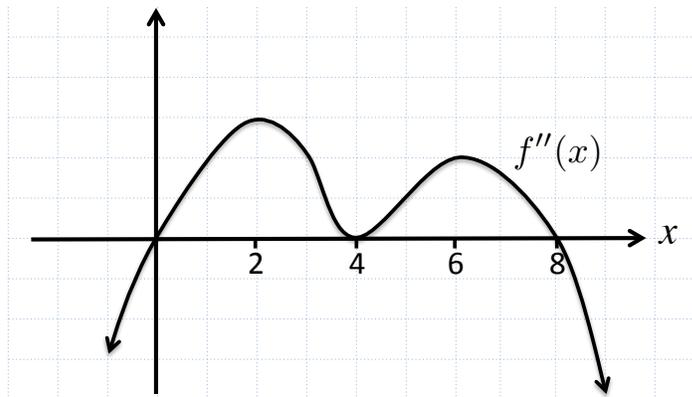
Answer:

ii. State the x -value(s) where a local minimum occurs.

Answer:

3 marks

(b) The graph of $f''(x)$ (the *second* derivative of $f(x)$) is displayed below. For what x -value(s) does $f(x)$ have an inflection point?



Answer:

Full-solution problems: Justify your answers and **show all your work**. Place a box around your final answer. Unless otherwise indicated, **simplification of answers are required in these questions**.

10 marks

7. Let $f(x) = \frac{1}{2x+1}$. Use the definition of the derivative to find $f'(2)$. No marks will be given for the use of any differentiation rules.

10 marks

8. A company is planning to produce a new electric toaster. After conducting an extensive market survey, the research department provides the following information: when the unit price is \$16 per toaster, then the weekly demand is 20 toasters; for every \$2 decrease in the unit price, the weekly demand increases by 10 toasters. The financial department estimate that the weekly fix costs will be \$140 and the variable cost of production will be \$4 per toaster. Let p be the unit price and let q be weekly demand. Note: *Please simplify your answers such that your answers for (b) and (c) are given by integers.*

- (a) Find the demand function linking p and q .

Answer:

- (b) What price should the company charge in order to maximize the weekly profit?

Answer:

- (c) Find the break-even points for the company.

Answer:

10 marks

9. A carpenter has been asked to build an open box with a square base, where an open box means a box without a top. The sides of the box will cost \$2.50 per square meter and the base will cost \$5 per square meter. What are the dimensions of the box of maximal volume that can be constructed for \$60? Note: *In order to receive full credit, you will need to justify your answer.*

Answer:

12 marks

10. Let $f(x) = x^2 + x - \frac{1}{x} - 1$. Its derivative satisfies

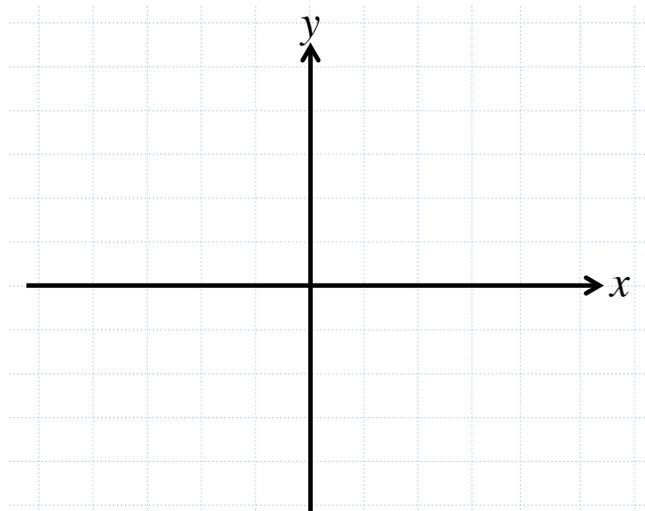
$$f'(x) = 2x + 1 + \frac{1}{x^2} = \frac{2x^3 + x^2 + 1}{x^2} = \frac{(x+1)(2x^2 - x + 1)}{x^2}.$$

(a) Evaluate $\lim_{x \rightarrow 0^-} f(x)$ and $\lim_{x \rightarrow 0^+} f(x)$.

(b) Find the intervals on which f is increasing or decreasing.

(c) Find the intervals on which f is concave up. Hint: think about what version of $f'(x)$ you would like to differentiate!

(d) Sketch the graph of f . Note: Your graph should be reasonably accurate within the grid, so you may not use the whole x-axis).



10 marks

11. A spotlight on the ground shines on a wall 15 m away. If a woman 2 m tall walks from the spotlight toward the wall at a speed of 0.7 m/s, how fast is the length of her shadow (on the building) changing when she is 8 m from the building? State your answer accurate to 2 decimal places.

Answer:

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Do not remove it!*