

The University of British Columbia

Final Examination - December 6, 2014

Mathematics 104/184

All Sections

Closed book examination

Time: 2.5 hours

Last Name _____ First _____ Signature _____

MATH 104 or MATH 184 (Circle one) Student Number _____

Instructor's Name _____ Section Number _____

Special Instructions:

No books, notes, or calculators are allowed.

Student Conduct during Examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)-(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

| | | |
|-------|--|-----|
| 1 | | 42 |
| 2 | | 9 |
| 3 | | 15 |
| 4 | | 10 |
| 5 | | 17 |
| 6 | | 7 |
| Total | | 100 |

[42] **1. Short Problems.** Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{3 - x}$.

Answer:

(b) Evaluate $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16}$.

Answer:

(c) Let $f(x) = \begin{cases} 3x & \text{if } x \leq 2 \\ x^2 - 2x & \text{if } x > 2 \end{cases}$. Is f continuous from the right at 2?

Answer:

- (d) Let $f(x) = \frac{x^3}{3} + x^2 - 7x + 100$. Find the point on the graph of f at which the tangent line has slope -8 .

Answer:

- (e) Suppose $f(4) = 3$ and $f'(4) = 2$. Let $h(x) = \frac{f(x)}{x-5}$. Find an equation of the line tangent to $y = h(x)$ at $x = 4$.

Answer:

- (f) Find $\frac{dy}{dx}$, where $y = e^{9x} \cos x$.

Answer:

- (g) Suppose the position of an object moving horizontally after t seconds is given by the function $s = f(t) = t^4 - 2t$ for $0 \leq t \leq 6$. Find the acceleration of the object at $t = 2$.

Answer:

- (h) Assume f and g are differentiable on their domains with $h(x) = f(g(x))$. Suppose the equation of the line tangent to the graph of g at the point $(4, 7)$ is $y = 3x - 5$ and the equation of the line tangent to the graph of f at $(7, 9)$ is $y = -2x + 23$. Find $h'(4)$.

Answer:

- (i) Find $\frac{dy}{dx}$, where $y = x^{\ln x}$.

Answer:

- (j) An investment earns at an annual interest rate of 5% compounded continuously. How fast is the investment growing when its value is \$500?

Answer:

- (k) Find $f'(0)$, where $f(x) = \tan^{-1}(5x)$. Note that \tan^{-1} refers to the inverse tangent function, which is also denoted by \arctan .

Answer:

- (l) Suppose f is differentiable on $(-\infty, \infty)$ and assume it has a local extreme value at the point $x = 2$, where $f(2) = 0$. Let $h(x) = xf(x) + 2x + 3$ for all values of x . Does $h(x)$ have a local extreme value at $x = 2$?

Answer:

(m) Estimate $|\sin(0.12) - 0.12|$ by considering the linear approximation of $\sin x$ at $x = 0$.

Answer:

(n) Let $f(x) = x^{2/3}$. Find the second-order Taylor polynomial for $f(x)$ with its center at 1.

Answer:

Long Problems. In questions 2–6, show your work. No credit will be given for the answer without the correct accompanying work.

[9] **2.** Let $f(x) = \frac{x^2}{x^2 + 1}$. Use the definition of the derivative to find $f'(1)$. No marks will be given for the use of any differentiation rules.

[15] **3.** The price p (in dollars) and the demand q for a product are related by the following demand equation:

$$p^3 + q + q^3 = 38.$$

(a) Find the price elasticity of demand in terms of p and q for this product.

(b) If the current demand is $q = 3$, will the revenue increase or decrease if the price is raised slightly?

c) Suppose the price increases at a rate \$7/month. How fast does the demand decrease when the demand is $q = 3$?

[10] 4. You are planning a city tour for a group of 100 tourists. If you can sell x bus tour tickets, you can offer them for $\$(30 - x/4)$ each. If you can sell y boat tour tickets, you can offer them for $\$(70 - y/2)$ each. How many bus tickets, and how many train tickets should you sell to the tourists in order to maximize revenue (you can only sell one type of ticket to each passenger).

[17] 5. Let $f(x) = \frac{e^x}{x^2}$.

(a) Find the critical point of $f(x)$.

(b) Find the intervals on which f is increasing or decreasing.

(c) Find $f''(x)$.

(d) Find the intervals on which f is concave upward.

(e) Find the horizontal asymptote and the vertical asymptote of $f(x)$. Note that $\lim_{x \rightarrow \infty} \frac{e^x}{x} = \infty$.

(f) Sketch the graph of $f(x) = \frac{e^x}{x^2}$.

[7] **6.** Let $f(x) = \frac{x^4}{12} - \frac{4(3^{-x})}{(\ln 3)^2} + \log_{(x+1)^2}(x+1)^{2014}$ be a function defined on the interval $(0, \infty)$. Show that the graph of $f(x)$ on the interval $(0, \infty)$ has an inflection point.