MATHEMATICS 184 Section 201
Final Exam, April 22, 2013, 8:30 - 11 am

Last Name: **First Name:** **Student No.:**

Q1		16
Q2		8
Q3		8
Q4		10
Q5		10
Q6		8
Q7		10
Q8		10
Total		80

Note: This test has 10 pages including the title page. Problem 1 requires a short answer for each part, and full marks will be given for a correct answer in the box. No credit is given for an incorrect answer. The others are long-answer questions, and you should show all your work and simplify your results.

Rules of Conduct

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBC card for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in the questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   - speaking or communicating with other examination candidates, unless otherwise authorized;
   - purposely exposing written papers to the view of other examination candidates or imaging devices;
   - purposely viewing the written papers of other examination candidates;
   - using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   - using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) — (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1. (16 marks, 2 for each part.) The problem consists of 8 short-answer questions:

(a) Evaluate $\lim_{x \to 2} \frac{\sqrt{x^2+12}-x}{x-1}$ or determine that it does not exist. In the latter case, write DNE as answer.

Answer

(b) Find values of a that would make the following function continuous everywhere, or determine that no such value exists. In the latter case, write DNE as answer.

$$g(x) = \begin{cases} \sqrt{x^2+2} & \text{if } x \le 0 \\ x+1 & \text{if } 0 < x \le 1 \\ ax^3 & \text{if } x > 1. \end{cases}$$

Answer

(c) Find the derivative of $y = x^x \ln x$.

Answer

(d) Find the equation for the tangent line to the curve $x^2y + xy^2 = 2$ at (1,1).

Answer

(e) Use linear approximation to estimate $(0.998)^3$.

Answer

(f) Find the vertical, horizontal or slant asymptotes of $f(x) = \dfrac{x^2 - 4}{x + 1}$ if any exists. Write DNE if none exists.

Answer

(g) Find any local maxima and minima of $h(x) = x^{-2}e^x$ in the interval $[1, 4]$. Give both x and $h(x)$ values and state whether it is a local minimum or maximum.

Answer

(h) On which part of the interval $[0, \pi]$ is $f(x) = e^x \sin x$ concave upward?

Answer

2. (8 marks) Let $g(x) = \begin{cases} 4x^2 + 2x + 1 & \text{if } x \leq 0 \\ \sqrt{4x+1} & \text{if } x > 0 \end{cases}$

(a) Is the function continuous at $x = 0$? Justify your answer.

(b) Use the definition of derivative to find $g'(0)$ or show that it does not exist.

3. (8 marks) A function $y(x)$ obeys this relationship: $y' = xy^2 + ax^2$, where a is a constant.

(a) Determine the value of a such that the tangent line to the graph of $y(x)$ at $(1, 1)$ passes through $(2, 5)$.

(b) For this a value, determine the concavity of the function at $x = 1$.

4. (10 marks) Opad, the blockbuster product of Opple Inc., has a weekly demand q that declines with price p according to $q = 1000\,e^{-p/200}$.

   (a) Find the elasticity of demand ϵ at the current price of \$100.

   (b) Use the elasticity of demand to calculate the marginal revenue at the current price of \$100. Simplify your answer to "calculator ready".

   (c) If the price is raised by 1%, use the elasticity to estimate the percentage decline in sales.

5. (10 marks) A rectangular poster is to have an area of 150 cm^2 with 1-cm margins on the bottom and the sides, and a 2-cm margin at the top. What width and height of the poster will give the largest printed area?

6. (8 marks) Market research has shown that if promotion of a product is stopped and other market conditions remain unchanged, sales of this product decline at a rate that is proportional to the current sales. At the end of January, Champion Sports was selling 1,000 tennis rackets per day when they stopped promotion of the product. Two months later, the sales of rackets have come down to 600 per day. What will the sales be in another 4 months?

7. (10 marks) A ladder initially leans against a vertical wall, with its bottom resting on flat ground. Then the bottom starts to slide away from the wall at a rate of 2 m/min. At the moment that the top of the ladder is 4 m above the ground, it is sliding down the wall at 1.5 m/min. What is the length of the ladder?

8. (10 marks)

(a) Write out the third-order Maclaurin polynomial for the function $\ln(1+x)$.

(b) How many terms in the Maclaurin polynomial should you keep if you wish to approximate $\ln(1.1)$ with an error below 10^{-6}?