The University of British Columbia

Final Examination - April 25, 2014

Mathematics 101

Time: 2.5 hours

FAMILY Name ____________________

First Name ____________________ **Signature** ____________________

Student Number ____________________

MATH 101 **Section Number:**____________

Special Instructions:

No memory aids are allowed. No communication devices allowed. No calculators allowed. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them. Calculator-ready form is acceptable for all numerical answers.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		33
2		6
3		6
4		6
5		6
6		5
7		6
8		6
9		6
Total		80

[33] **1**. **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Evaluate $\int x \ln x \, dx$.

Answer:

(b) Evaluate $\int \frac{x^2}{(x^3+1)^{101}} \, dx$.

Answer:

(c) Evaluate $\int \cos^3 x \sin^4 x \, dx$.

Answer:

(d) Evaluate $\int \sqrt{4-x^2}\,dx$.

Answer:

(e) Consider the Trapezoid Rule for making numerical approximations to $\int_a^b f(x)\ dx$. The error for the Trapezoid Rule satisfies $|E_T| \leq \dfrac{K(b-a)^3}{12n^2}$, where $|f''(x)| \leq K$ for $a \leq x \leq b$. If $-2 < f''(x) < 0$ for $1 \leq x \leq 4$, find a value of n to guarantee the Trapezoid Rule will give an approximation for $\int_1^4 f(x)\ dx$ with absolute error, $|E_T|$, less than 0.001.

Answer:

(f) Find the values of p for which the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges.

Answer:

(g) Find the radius of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$.

Answer:

(h) Find a power series representation for $\dfrac{x^3}{1-x}$.

Answer:

(i) Find the coefficient c_5 of the fifth degree term in the Maclaurin series $\sum_{n=0}^{\infty} c_n x^n$ for e^{3x}.

Answer:

(j) Let $f(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2}\, dt$. Find the interval(s) on which f is increasing.

Answer:

(k) Use series to evaluate $\lim_{x \to 0} \frac{1 - \cos(x)}{1 + x - e^x}$.

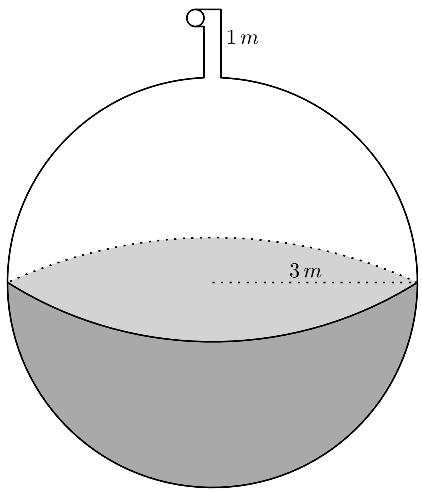
Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[6] **2**. Find the area of the finite region bounded between the two curves $y = \sqrt{2}\cos(\pi x/4)$ and $y = |x|$. It will be useful to sketch the region first.

[6] **3**. Find the volume of the solid generated by rotating the finite region bounded by $y = 1/x$ and $3x + 3y = 10$ about the x-axis. It will be useful to sketch the region first.

[6] **4**. A spherical tank of radius 3 metres is half-full of water. It has a spout of length 1 metre sticking up from the top of the tank. Find the work required to pump all of the water in the tank out the spout. The density of water is 1000 kilograms per cubic metre. The acceleration due to gravity is 9.8 metres per second squared.



[6] **5**. Find the centroid of the finite region bounded by $y = \sin(x)$, $y = \cos(x)$, $x = 0$, and $x = \pi/4$.

[5] **6**. Find the solution of the differential equation

$$x\frac{dy}{dx} + y = y^2$$

that satisfies $y(1) = -1$.

[6] **7**. Determine, with explanation, whether the following series converge or diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{2^n}$

[6] **8**. Find the Taylor series for $f(x) = \ln(x)$ centred at $a = 2$. Find the interval of convergence for this series.

[6] **9**. (a) Show that $\sum_{n=0}^{\infty} nx^n = \dfrac{x}{(1-x)^2}$ for $-1 < x < 1$.

(b) Express $\sum_{n=0}^{\infty} n^2 x^n$ as a ratio of polynomials. For which x does this series converge?