

Marks

- [42] 1. **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a) Evaluate $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1}$ or determine that this limit does not exist.

Answer

(b) Evaluate $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x)$ or determine that this limit does not exist.

Answer

(c) Find the derivative of $f(x) = x \cos x$.

Answer

- (d) Find the derivative of $\frac{e^x}{x^2 - 3}$.

Answer

- (e) Find the derivative of $y = \sin^5(\cos(3x^2))$.

Answer

- (f) Find the derivative of $y = \sin^{-1}(\sqrt{x})$. [Note: Another notation for \sin^{-1} is arcsin.]

Answer

- (g) Find the slope of the tangent line to the curve $x^4 - x^2y + y^4 = 1$ at the point $(-1, 1)$.

Answer

- (h) Find $f'(x)$, if $f(x) = (\tan x)^{\cos x}$.

Answer

- (i) Use a suitable linear approximation to estimate $(30)^{1/3}$. Give your answer as a fraction with integer numerator and denominator.

Answer

- (j) Find the second degree Taylor polynomial $T_2(x)$ for $f(x) = \sqrt{x}$ at 4 (or about $x = 4$).

Answer

- (k) If $f(0) = 10$ and $f'(x) \geq 3$ for $0 \leq x \leq 4$, what is the least $f(4)$ could possibly be?

Answer

- (l) Find the absolute maximum *value* of $f(x) = e^x(\sin x - \cos x)$ on the interval $[\frac{\pi}{2}, 2\pi]$.

Answer

- (m) Newton's Method is used to approximate a solution of the equation $e^x - x^2 = 0$, starting with the initial approximation $x_1 = 0$. Find x_3 .

Answer

- (n) Find $f(t)$, if $f''(t) = \frac{3}{\sqrt{t}}$, $f(4) = 2$ and $f'(4) = 7$.

Answer

Full-Solution Problems. In questions 2–8, justify your answers and **show all your work**. If a box is provided, write your final answer there. Simplification of answers is not required unless explicitly requested.

- [9] 2. When an apple is taken from a refrigerator, its temperature is 3°C . After 30 minutes in a 19°C room, its temperature is 11°C .

(a) Assuming that the temperature of the apple satisfies Newton's Law of Cooling, write the *differential equation* satisfied by the temperature $T(t)$ of the apple at time t .

Answer

(b) Find the temperature of the apple 90 minutes after it is taken from the refrigerator, expressed as an integer number of degrees Celsius.

Answer

(c) Determine when the temperature of the apple is 16°C .

Answer

- [9] **3.** Two cylindrical swimming pools are being filled simultaneously with water, at exactly the same rate measured in m^3/min . The smaller pool has a radius of 5 m and the height of the water in smaller pool is increasing at a rate of 0.5 m/min. The larger pool has a radius of 8 m. How fast is the height of the water increasing in larger pool? Your answer must be a specific numerical value.

Answer

[14] 4. Let

$$f(x) = \begin{cases} \frac{4}{\pi} \tan^{-1} x, & \text{if } x \geq 1, \\ 2 - x^4, & \text{if } x < 1. \end{cases}$$

[*Note:* Another notation for \tan^{-1} is \arctan .]

(a) (3 marks) Show that $f(x)$ is continuous at $x = 1$.

(b) (1 mark) Determine the equations of any asymptotes (horizontal, vertical or slant).

(c) (4 marks) Determine all critical numbers, open intervals where f is increasing or decreasing, and the x -coordinates of all local maxima or local minima (if any).

Question 4 continues on the next page...

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Question 4 continued

- (d) (2 marks) Determine open intervals where the graph of f is concave upwards or concave downwards, and the x -coordinates of all inflection points (if any).
- (e) (4 marks) Sketch the curve $y = f(x)$, showing all the features given in items (a) to (d) above and giving the (x, y) coordinates for all points occurring above (if any).

- [10] 5. Cobblestone Engineering has been contracted to build a bridge and pathway from Calculus City located at point A on the shoreline of a 6 km wide river that runs west to east, to a power plant at point B on the opposite shoreline, 8 km to the east. It costs 40 dollars per km to build a bridge over the river and 20 dollars per km to build a pathway along a shoreline. How should the company proceed in order to minimize the total cost? Be sure to justify that the total cost is indeed a minimum.

Answer

- [4] **6.** Find the derivative of the function

$$\frac{t}{t+5}$$

using the definition of the derivative. [Note: No credit will be given for using differentiation rules such as the Power Rule, Product Rule, Quotient Rule, or Chain Rule, although you may use these rules to check your answer.]

- [6] 7. Determine and carefully justify an upper bound for the absolute value of the *error* that would result if the second degree Maclaurin polynomial $T_2(x)$ was used to estimate $f(1)$, where

$$f(x) = e^x(x^2 - 7x + 15).$$

Your upper bound may be left unsimplified (“calculator-ready”) but it must be a specific numerical value. [*Note:* You are *not required* to find $T_2(x)$, and no credit will be given for finding $T_2(x)$.]

- [6] 8. Two points on the surface of the Earth are called *antipodal* if they are at exactly opposite points (for example, the North Pole and South Pole are antipodal points). Prove that, at any given moment, there are two antipodal points on the equator with exactly the same temperature. *Hint:* Let $T(\theta)$ be the temperature, at any given moment, at the point on the equator with longitudinal angle θ measured in radians, $0 \leq \theta \leq 2\pi$ (i.e. in one complete trip around the equator, θ goes from 0 to 2π), and consider $f(\theta) = T(\theta + \pi) - T(\theta)$.

Be sure that this examination has 13 pages including this cover

The University of British Columbia
Sessional Examinations - December 2010

Mathematics 100/180

Differential Calculus with Applications to Physical Sciences and Engineering

Closed book examination

Time: 2.5 hours

Surname(s): _____ Given Name(s): _____

Student Number: _____ Instructor's Name: _____

Signature: _____ Section Number: _____

Special Instructions:

No books, notes, or calculators are allowed. Unless otherwise specified, answers that are numerical constants may be left in “calculator-ready” form, where calculator means basic scientific calculator. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

1. Each candidate should be prepared to produce, upon request, a UBCcard for identification.
2. No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
3. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices.The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.
7. Smoking is not permitted during examinations.

1		42
2		9
3		9
4		14
5		10
6		4
7		6
8		6
Total		100