Be sure that this examination has 10 pages including this cover

The University of British Columbia

Sessional Examinations - December 2005

Mathematics 100/180

Differential Calculus with Applications to Physical Sciences and Engineering

Closed book examination | Time: 2.5 hours

Print Name ______________________ **Signature** ______________________

Student Number ______________________ **Instructor's Name** ______________________

Section Number ______________________

Special Instructions:

No calculators, cell phones, notes, or books of any kind are allowed. Show all calculations for your solutions. If you need more space than is provided, use the back of the previous page.

Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

1. Each candidate should be prepared to produce his or her library/AMS card upon request.
2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		33
2		10
3		12
4		8
5		15
6		12
7		10
Total		100

Marks

[33] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a) Evaluate

$$\lim_{x\to 9} \frac{x\left(\sqrt{x}-3\right)}{x-9}.$$

Answer

(b) Calculate and completely simplify the derivative of $f(x) = \ln\left(1000\tan^{-1}(x)\right)$.
[*Note*: Another notation for $\tan^{-1}$ is arctan].

Answer

(c) Find an equation of the line tangent to the graph of $x^3 + y^3 = 3xy$ at the point $(x, y) = (3/2, 3/2)$.

Answer

Continued on page 3

(d) Find the *second* derivative of $g(x) = \sin\left(e^{2x}\right)$.

Answer

(e) If we expand $\sin^2(x)$ in a Maclaurin series, say $\sin^2(x) = \sum_{n=0}^{\infty} c_n x^n$, find c_6.
[*Hint:* $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$].

Answer

(f) Find $f'(x)$ if $f(x) = (\sin x)^x$.

Answer

(g) Suppose we know that $f(1) = f'(1) = 1$. Define $g(x) = f(x^3)$. Use a linear approximation to the function g (*not* a linear approximation to the function f) to estimate $g(1.1)$.

Answer

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(h) Use a suitable linear approximation to estimate $(17)^{1/4}$. Give your answer as a fraction with integer numerator and denominator.

Answer

(i) What does Taylor's Inequality give as an upper bound to the error in part (h)?

Answer

(j) Using Newton's Method, starting with $x_1 = 2$, find the approximation x_3 to the root of the equation $20x - x^3 - 24 = 0$.

Answer

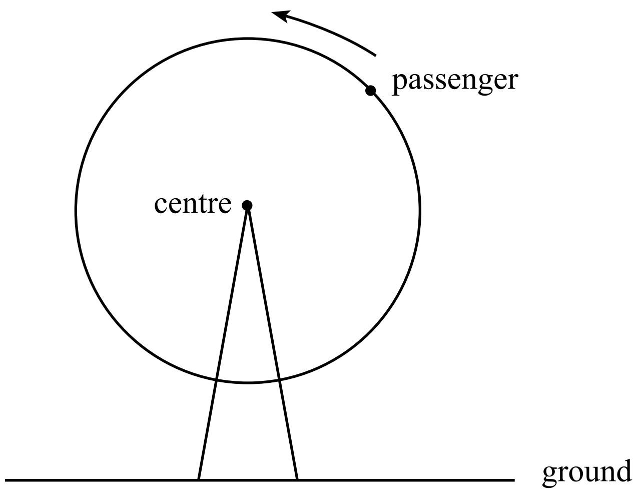
(k) A particle moves in a straight line so that its velocity at time t is $v(t) = \sqrt{t}$. If its position at time 9 is $s(9) = 20$, find $s(10)$.

Answer

Continued on page 5

Full-Solution Problems. In questions 2–7, justify your answers and **show all your work**. If a box is provided, write your final answer there. Unless otherwise indicated, simplification of answers is not required.

[10] **2.** A circular ferris wheel with radius 10 metres is revolving at the rate of 10 radians per minute. How fast is a passenger on the wheel rising when the passenger is 6 metres higher than the centre of the wheel and is rising? Include units in your answer.



Answer

Continued on page 6

[12] **3.** Wallapak stores her malt beverages outside her house, on her back porch. On a very hot day, she takes one such beverage and places it in her refrigerator, which is constantly kept at 4°C. After 15 minutes, the beverage cooled down to 26°C, and, after another 15 minutes, it was at 15°C. How hot was the beverage when it was placed in the refrigerator? Assume the temperature of the beverage obeys Newton's Law of Cooling.

Answer

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[8] **4.** Find the derivative of $f(x) = \sqrt{1-2x}$ *using the definition of derivative.* No credit will be given for using differentiation rules, but you may use differentiation rules to check your answer.

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[15] **5.** Let $f(x) = \dfrac{x}{3+x^2}$.

(a) Determine all of the following if they are present:

(i) (4 marks) critical numbers, x-coordinates of local maxima and minima, intervals where $f(x)$ is increasing or decreasing;

(ii) (4 marks) x-coordinates of inflection points, and intervals where $f(x)$ is concave upwards or downwards;

(iii) (2 marks) equations of any asymptotes (horizontal, vertical, or slant).

(b) (5 marks) Sketch the graph of $y = f(x)$, giving the (x, y) coordinates for all points of interest above. Draw your sketch on the back of the previous page.

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[12] **6.** At noon, a sailboat is 20 km due south of a freighter. The sailboat is traveling due east at 20 km/h, and the freighter is traveling due south at 40 km/h.

(a) (9 marks) When are the two ships closest to one another? Remember to justify your answer.

Answer

(b) (3 marks) If the visibility at sea is 10 km, could the people on the two ships ever see each other?

Answer

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[10] **7.** (a) (5 marks) A function $f(x)$ is defined to equal $\cos(ax) + b$ for $x \geq 0$ and $2 - x^3$ for $x < 0$, where a and b are constants. It is known that this function is differentiable everywhere. Find all possible values for a and b.

(b) (5 marks) A function $g(x)$ satisfies $g(0) = 0, g(1) = 1$, and $g(2) = -1$. It is known that this function is twice differentiable everywhere (i.e. $g''(x)$ exists for all x). Prove that $g'(c) = 0$ for some real number c. Give complete justification, specifying any relevant theorems.

The End