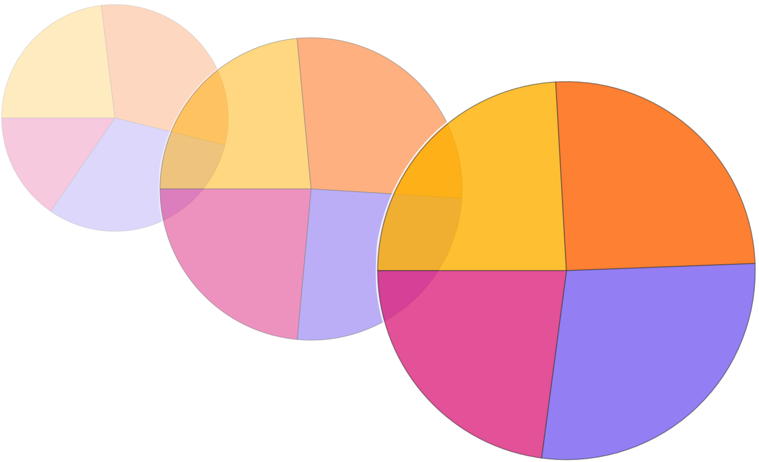
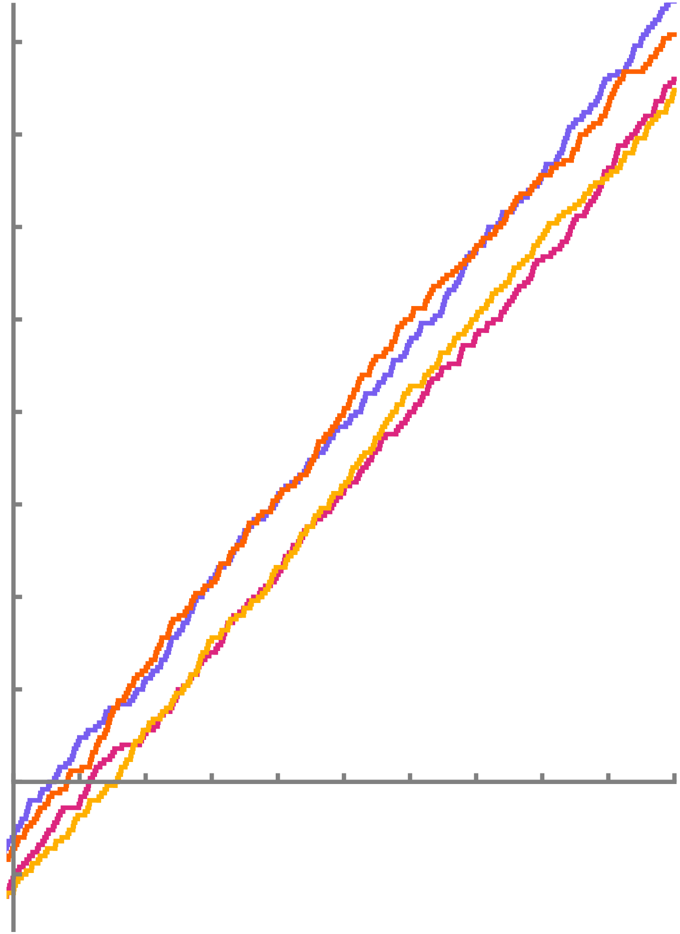
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |





# Last-digit prime number races

Greg Martin
University of British Columbia, Department of Mathematics

*April 9, 2025.* Every prime number other than 2 and 5 has its last digit equal to **1**, **3**, **7**, or **9**, dividing the primes into four "teams". Each of these teams has infinitely many primes; how soon the primes for each team occur can be counted using the functions

$$\pi(x; 10, a) = \#\{\text{primes } p \colon p \le x, \text{the last digit of } p \text{ is } a\}$$

for $a \in \{\mathbf{1}, \mathbf{3}, \mathbf{7}, \mathbf{9}\}$. For any given value of $x$, these four counting functions form a pie chart that indicates which teams of primes are more or less successful to that point.

The prime number theorem for arithmetic progressions implies that the four teams are, in the limit, equally successful—the quotient of any two of those counting functions tends to 1 as $x \to \infty$. So as $x$ becomes larger and larger, the pie chart looks more and more like an even division into four quarters.

Nevertheless, the teams with last digits **3** and **7** seem to be more successful than the teams with last digits **1** and **9** for most individual values of $x$. Comparing graphs of the step functions $\pi(x; 10, a)$, most of the time Teams **3** and **7** are competing for first and second place, while Teams **1** and **9** are competing for third and fourth place. In fact, analysis suggests that this is the case 84.54% of the time [1].

The field of comparative prime number theory studies this phenomenon and analogues where the arithmetic progressions have common differences other than 10 [2]. A twelve-author UBC team assembled an annotated bibliography for the subject [3].

[1] Feuerverger & Martin. Biases in the Shanks–Rényi prime number race. *Exp. Math.* 2000.
[2] Granville & Martin. Prime number races. *Am. Math. Mon.* 2006.
[3] Martin *et al.* An annotated bibliography for comparative prime number theory. *Expo. Math.* 2025.