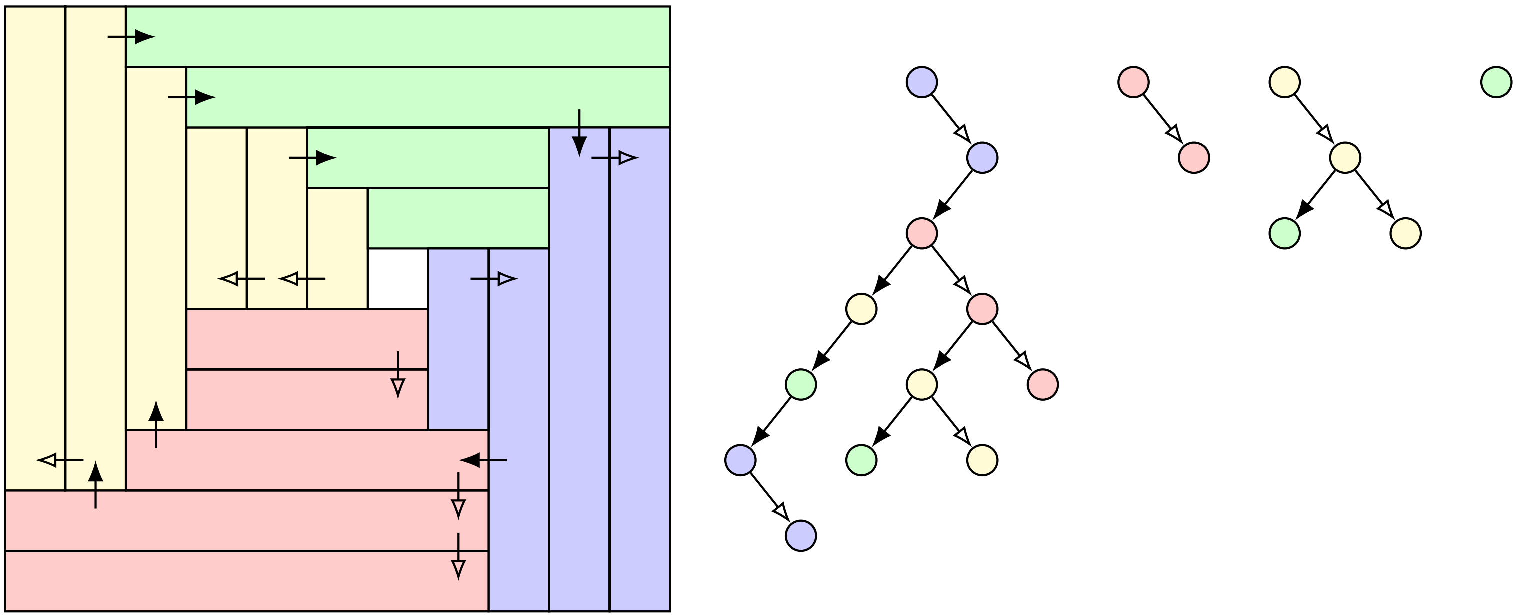

Counting simple whirls

Hugh Thomas
Université du Québec à Montréal, Département de mathématiques

February 11, 2025. A rectangulation decomposes a rectangle into subrectangles, no four meeting at a point. Rectangulations are considered up to a natural combinatorial equivalence.

Patterns arise in a rectangulation, for example, there is an occurrence of a windmill  in the image obverse, centered at the blank square. A *whirl* is a rectangulation that avoids patterns

and a whirl is *peelable* if it contains a rectangle that stretches from one boundary to the opposite one. A *simple* whirl is a non-peelable whirl containing exactly one  whose interior is not further divided.

The generating function $t^5C^4(t)$ counts simple whirls [1], where $C(t)$ is the well-known Catalan generating function

$$C(t) = \frac{1 - \sqrt{1-4t}}{2t}$$

which counts, in particular, planar binary trees.

The image obverse shows how a simple whirl can be decomposed into four planar binary trees, each of whose roots has no left child. The nodes on the right correspond to the rectangles on the left. This gives a bijective proof of the result of Asinowski and Banderier, and is part of a joint project-in-progress with those authors.

[1] Asinowski & Banderier. From geometry to generating functions: rectangulations and permutations. Sém. Loth. Comb., 2024