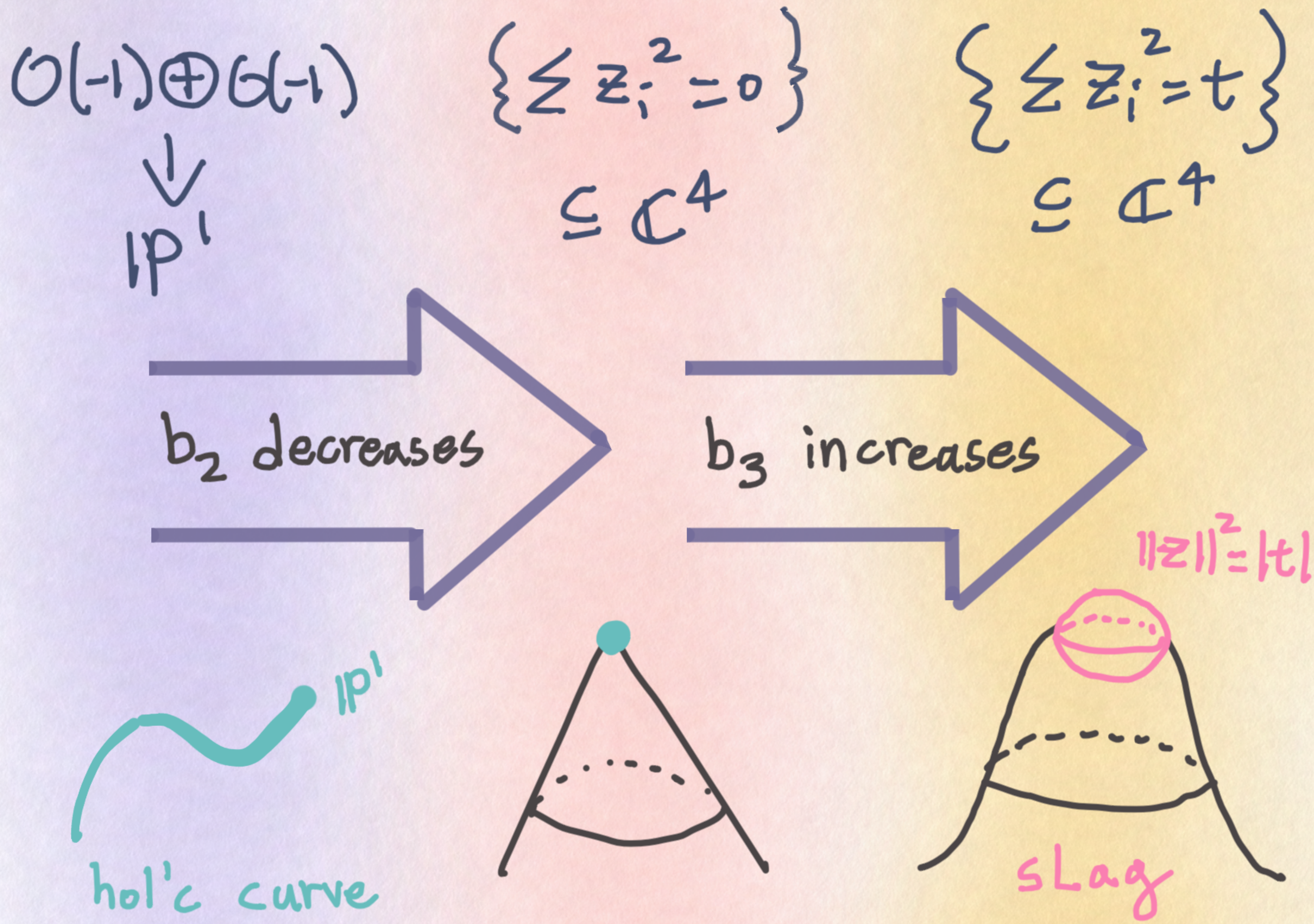

$\mathcal{O}(-1)\oplus\mathcal{O}(-1)$
$\downarrow$
$\mathbb{P}^1$
$\{\sum z_i^2 = 0\}$
$\subseteq \mathbb{C}^4$
$\{\sum z_i^2 = t\}$
$\subseteq \mathbb{C}^4$
b_2 decreases
b_3 increases
$\|z\|^2 = |t|$
$\mathbb{P}^1$
hol'c curve
sLag

Transitions & Fluxes

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September 21, 2022. Maxwell's equations in a vacuum satisfy

$$dF = 0, \quad d^*F = 0,$$

where F is a 2-form—sometimes called the electromagnetic field strength—in Minkowski space-time $\mathbb{R}^{3,1}$. In the presence of electric charge one of these equations is modified to $d^*F = j_E$ and, by Gauss's law, the electric flux through a 2-sphere is $Q_E = \int_{S^2} \star F$. However, non-zero magnetic flux through a closed surface has never been observed in nature, and this creates an intriguing asymmetry between electric and magnetic fields. The possibility of electromagnetic duality led physicists to consider the hypothetical magnetic monopole equation $dF = j_M$ with net magnetic charge $Q_M = \int_{S^2} F$.

There is a string theory analog of these equations [1] involving a 3-form field strength H, which in a vacuum satisfies the equations

$$dH = 0, \quad d^*H = 0.$$

In this setup, extra dimensions are introduced by considering the product $\mathbb{R}^{3,1} \times X$. Starting from principles of supersymmetry and a warped product ansatz [2], physicists reduce their equations from the product to the internal space X, deducing that X is a compact complex three-dimensional manifold with hermitian metric ω, holomorphic volume form Ω, and H takes the form $H = i(\partial - \bar{\partial})\omega$. Kähler Calabi-Yau threefolds (X, ω, Ω) are those with $H = 0$.

We now apply these ideas to Calabi-Yau conifold transitions. A conifold transition is a mechanism for passing from one Calabi-Yau threefold to another by degenerating 2-cycles and introducing new 3-cycles—see the front of the postcard for the local model. This procedure alters Betti numbers, and allows travel in the parameter space of Calabi-Yau threefolds. By degenerating 2-cycles, it may happen that the resulting manifold has zero second Betti number, and thus cannot admit a Kähler metric. An initial Kähler Calabi-Yau threefold can then become a non-Kähler complex manifold through this process.

A conifold transition exchanges holomorphic 2-cycles C_i with special Lagrangian 3-spheres L_i [4], all with respect to the non-Kähler geometry (X, ω) from [3] solving $d\omega^2 = 0$. The corresponding 3-form field H satisfies $d^*H = 0$, but $dH \neq 0$, and the emerging special Lagrangian 3-spheres have non-zero enclosed charge $Q_M = \int_{S^3} H$. Thus the non-Kähler geometry (X, ω) used to study conifold transitions can be interpreted as a geometry with magnetic flux.

[1] Callan, Harvey & Strominger, Supersymmetric string solitons, *String theory and quantum gravity*, 1991.
[2] Strominger, Superstrings with torsion, *Nuclear Phys. B*, 1986.
[3] Fu, Li, & Yau, Balanced metrics on non-Kähler Calabi-Yau threefolds, *J. Differential Geom.*, 2012.
[4] Collins, Gukov, Picard & Yau, Special Lagrangian cycles and Calabi-Yau transitions, preprint.