On singularity of p-energy measures among distinct values of p for some p.-c.f. self-similar sets

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For each $p \in (1, \infty)$, a *p-energy form* $(\mathcal{E}_p, \mathcal{F}_p)$, a natural L^p-analog of the standard Dirichlet form for $p = 2$, was constructed on the (two-dimensional standard) Sierpiński gasket K by Herman–Peirone–Strichartz [Potential Anal. **20** (2004), 125–148]. As in the case of $p = 2$, it satisfies the *self-similarity (scale invariance)*

$$\mathcal{E}_p(u) = \sum_{j=1}^{3} \rho_p \mathcal{E}_p(u \circ F_j), \qquad u \in \mathcal{F}_p,$$

where $\{F_j\}_{j=1}^3$ are the contraction maps on $\mathbb{R}^2$ defining K through the equation $K = \bigcup_{j=1}^3 F_j(K)$ and $\rho_p \in (1, \infty)$ is a scaling factor determined uniquely by $(K, \{F_i\}_{i=1}^3)$ and p. While the construction of $(\mathcal{E}_p, \mathcal{F}_p)$ has been extended to general p.-c.f. self-similar sets by Cao–Gu–Qiu (2022), to Sierpiński carpets by Shimizu (2024) and Murugan–Shimizu (2024+) and to a large class of infinitely ramified self-similar fractals by Kigami (2023), very little has been understood concerning properties of important analytic objects associated with $(\mathcal{E}_p, \mathcal{F}_p)$ such as p-harmonic functions and p-energy measures, even in the (arguably simplest) case of the Sierpiński gasket.

This talk is aimed at presenting the result of the speaker's on-going joint work with Ryosuke Shimizu (Waseda University) that, *for a class of p.-c.f. self-similar sets with very good geometric symmetry, the p-energy measure $\mu^p_{\langle u \rangle}$ of any $u \in \mathcal{F}_p$ and the q-energy measure $\mu^q_{\langle v \rangle}$ of any $v \in \mathcal{F}_q$ are mutually singular for any $p, q \in (1, \infty)$ with $p \neq q$.* The keys to the proof are (1) new explicit descriptions of the global and local behavior of p-harmonic functions in terms of ρ_p, and (2) the highly non-trivial fact that *$\rho_p^{1/(p-1)}$ is strictly increasing in* $p \in (1, \infty)$, whose proof relies heavily on (1).