RANDOM WALKS ON GROUPS AND THE KAIMANOVICH-VERSHIK 1983 CONJECTURE FOR LAMPLIGHTER GROUPS

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JOINT WORK WITH RUSSELL LYONS

Let G be an infinite group with a finite symmetric generating set S. The corresponding Cayley graph on G has an edge between x, y in G if their ratio xy^{-1} is in S. Kaimanovich-Vershik (1983), building on fundamental results of Furstenberg, Derrienic and Avez, showed that G admits non-constant bounded harmonic functions iff the entropy of simple random walk on G grows linearly in time; Varopoulos (1985) showed that this is equivalent to the random walk escaping with a positive asymptotic speed. Kaimanovich and Vershik (1983) also described the lamplighter groups (groups of exponential growth consisting of finite lattice configurations) where (in dimension at least 3) the simple random walk has positive speed, yet the probability of returning to the starting point does not decay exponentially. They conjectured a complete description of the bounded harmonic functions on these groups. In dimension 5 and above, their conjecture was proved by Anna Erschler (2011). In the talk, I will discuss the background and present a proof of the Kaimanovich-Vershik conjecture for all dimensions, obtained in joint work with Russ Lyons; the case of dimension 3 is the most delicate. No prior knowledge of group theory will be assumed.