SCALING LIMITS AND CRITICAL BEHAVIOUR OF THE 4-DIMENSIONAL n-COMPONENT ϕ^4 SPIN MODEL

GORDON SLADE

The n-component ϕ^4 model is a ferromagnetic continuous-spin model with interesting critical behaviour. In particular, the one-component model is predicted to be in the same universality class as the Ising model. We study the n-component model on the 4-dimensional integer lattice, for all n greater than or equal to 1, with small coupling constant. We prove that the susceptibility has a logarithmic correction to mean field scaling, with exponent $(n+2)/(n+8)$ for the logarithm.

We also analyse the asymptotic behaviour of the pressure as the critical point is approached, and prove that the specific heat has fractional logarithmic scaling for $n = 1, 2, 3$; double logarithmic scaling for $n = 4$; and is bounded when $n > 4$. In addition, for the model defined on the 4-dimensional discrete torus, we prove that the scaling limit near the critical point is a multiple of the Gaussian free field on the continuum torus, whereas, in the subcritical regime, the scaling limit is Gaussian white noise with intensity equal to the susceptibility.

The proofs are based on a rigorous renormalisation group method.

This is joint work with Roland Bauerschmidt and David Brydges.