

SHARP BENEFIT-TO-COST RULES FOR THE EVOLUTION OF COOPERATION ON REGULAR GRAPHS

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We study two of the simple rules on finite graphs under the death-birth updating and the imitation updating discovered by Ohtsuki, Hauert, Lieberman, and Nowak [*Nature* **441** (2006) 502-505]. Each rule specifies a payoff-ratio cutoff point for the magnitude of fixation probabilities of the underlying evolutionary game between cooperators and defectors. We view the Markov chains associated with the two updating mechanisms as voter model perturbations. Then we present a first-order approximation for fixation probabilities of general voter model perturbations on finite graphs subject to small perturbation in terms of the voter model fixation probabilities. In the context of regular graphs, we obtain algebraically explicit first-order approximations for the fixation probabilities of cooperators distributed as certain uniform distributions. These approximations lead to a rigorous proof that both of the rules of Ohtsuki et al. are valid and are sharp.