THE SURVIVAL PROBABILITY AND r-POINT FUNCTIONS IN HIGH DIMENSIONS

REMCO VAN DER HOFSTAD

We investigate the survival probability, θ_n, in high-dimensional statistical physical models, where θ_n denotes the probability that the model survives up to time n. Models to which our results apply are oriented percolation above 4+1 dimensions, the contact process above 4+1 dimensions, and lattice trees above 10 dimensions. We show that, similarly to branching processes, for these models, Kolmogorov's result that $n\theta_n$ converges holds, as well as Yaglom's theorem stating that, conditionally on survival to time n, the number of particles is approximately n times an exponential random variable.

In more detail, we prove that if the r-point functions scale to those of the canonical measure of super-Brownian motion, and if a certain self-repellence condition is satisfied, then $n\theta_n \to 2/(AV)$, where

(a) A is the asymptotic expected number of particles alive at time n, and

(b) V is the vertex factor of the model.

Our proofs are based on simple weak convergence arguments.

In the case of oriented percolation, this reproves a result with den Hollander and Slade (that was proved using heavy lace expansion arguments), at the cost of losing explicit error estimates.

[This is joint work with Mark Holmes, building on work with Gordon Slade, Frank den Hollander and Akira Sakai.]