STOCHASTIC GEOMETRIC REPRESENTATIONS OF THE QUANTUM CURIE-WEISS MODEL

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We develop path integral representations for Quantum Ising models. To connect with classical Fortuin-Kasteleyn (FK) representation, we begin by presenting the FK representation of the classical Curie-Weiss model (the Ising model on complete graph) via the language of Poisson Point Processes. We then show how to derive a general FK representation for Quantum Ising model. This representation was originally developed by M. Campanino, A. Klein, J.F. Perez (1991) and M. Aizenman, A. Klein, C.M. Newman (1993). We apply the above to the quantum Curie-Weiss model in transversal field. First, we present the full FK representation of this model. Examining the form of the resulting measure and dropping the weight component from it leads to the natural extension of the Erdös - Rényi random graphs. Finally, we consider the ground state of the quantum Curie-Weiss model via partial FK representation. We prove the existence of a phase transition in the ground state when the strength of the transversal field equals one.