UNIFORMITY OF THE UNCOVERED SET OF RANDOM WALK AND CUTOFF FOR LAMPLIGHTER CHAINS

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We show that the threshold for a subset sampled uniformly from the range of a random walk on $\mathbb{Z}_n^d$, $d \geq 3$, to become indistinguishable from a uniformly chosen subset of $\mathbb{Z}_n^d$ is $1/2$ the cover time. As a consequence of our methods, we show that the total variation mixing time of the random walk on the lamplighter graph $\mathbb{Z}_2 \wr \mathbb{Z}_n^d$, $d \geq 3$, has a cutoff with threshold at $1/2$ the cover time. We give a general criterion under which both of these results hold; other examples for which this applies include bounded degree expander families, the intersection of an infinite super-critical percolation cluster with an increasing family of balls, the hypercube, and the Cayley graph of the symmetric group generated by transpositions. The proof also yields precise asymptotics for the decay of correlation in the uncovered set. This is joint work with Yuval Peres.