THE PHASE TRANSITION IN PERCOLATION ON THE HAMMING CUBE

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Consider percolation on the Hamming cube $\{0,1\}^n$ at the critical probability p_c (at which the expected cluster size is $2^{n/3}$). It is known that if $p = p_c(1+O(2^{-n/3}))$, then the largest component is of size roughly $2^{2n/3}$ with high probability and that this random variable is not-concentrated. We show that for any sequence $\varepsilon(n)$ such that $\varepsilon(n) \gg 2^{-n/3}$ and $\varepsilon(n) = o(1)$ percolation at $p_c(1+\varepsilon(n))$ yields a largest cluster of size $(2+o(1))\varepsilon(n)2^n$. This completes the description of the phase transition on the Hamming cube and settles a conjecture of Borgs, Chayes, van der Hofstad, Slade and Spencer.

Our approach is to show that large percolation clusters have inherent randomness causing them to clump together and form a giant cluster. The behavior of the random walker on the Hamming cube plays a key role in the proofs of such statements.

Joint work with Remco van der Hofstad.